Throughput Maximization on Identical Machines

Coauthors: Benjamin Moseley, Kirk Pruhs, Clifford Stein

IPCO 2022

Summary

- A deterministic $O(1)$-competitive alg. for Online Throughput Maximization on $m > 1$ machines.
- Concludes 20-year line of research since $m = 1$ case settled

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- A deterministic $O(1)$-competitive alg. for Online Throughput Maximization on $m > 1$ machines.
- Concludes 20-year line of research since $m = 1$ case settled
- **Algorithm:** Run 3 algs. on $m/3$ machines each

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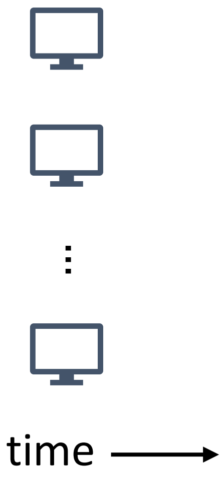
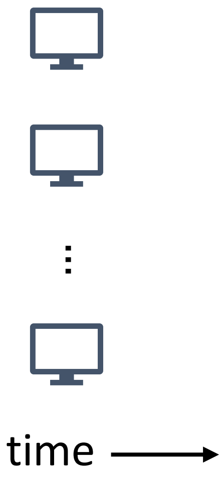
Online Throughput Maximization

- m identical machines



Online Throughput Maximization

- m identical machines
- Jobs arrive online at their release times with sizes and deadlines



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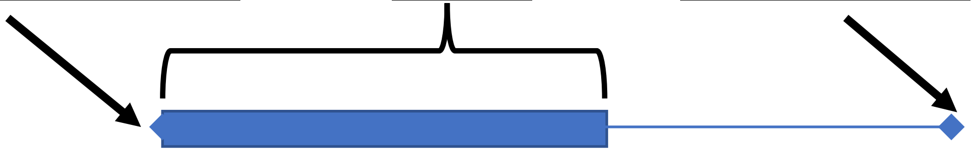

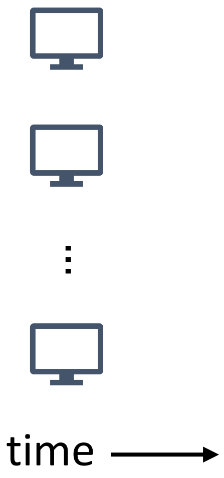

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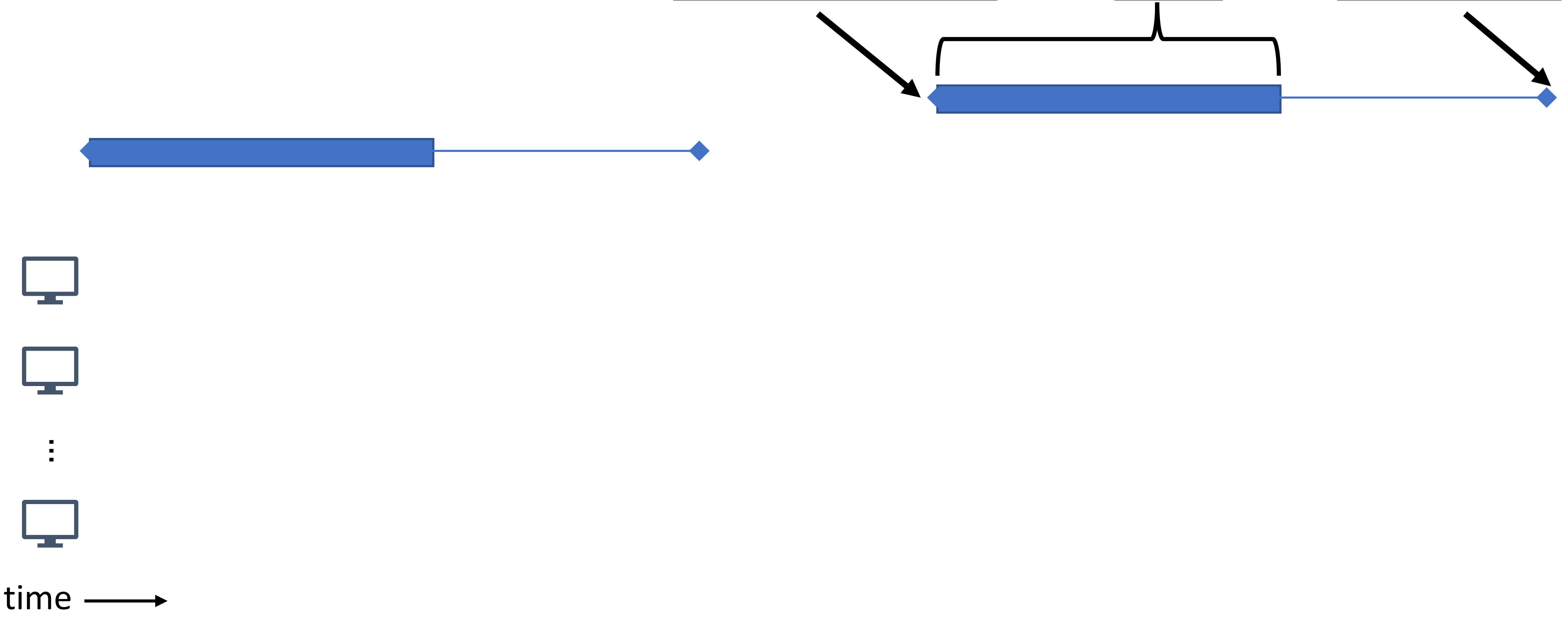

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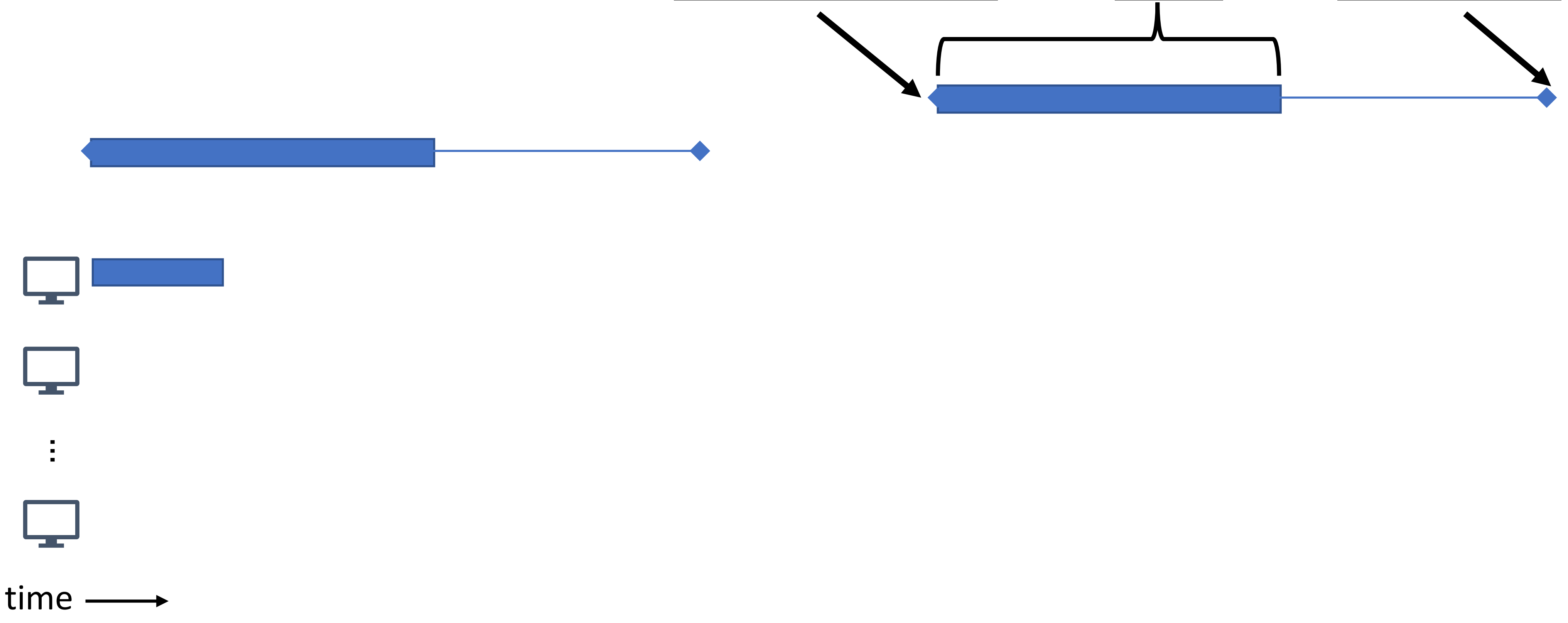

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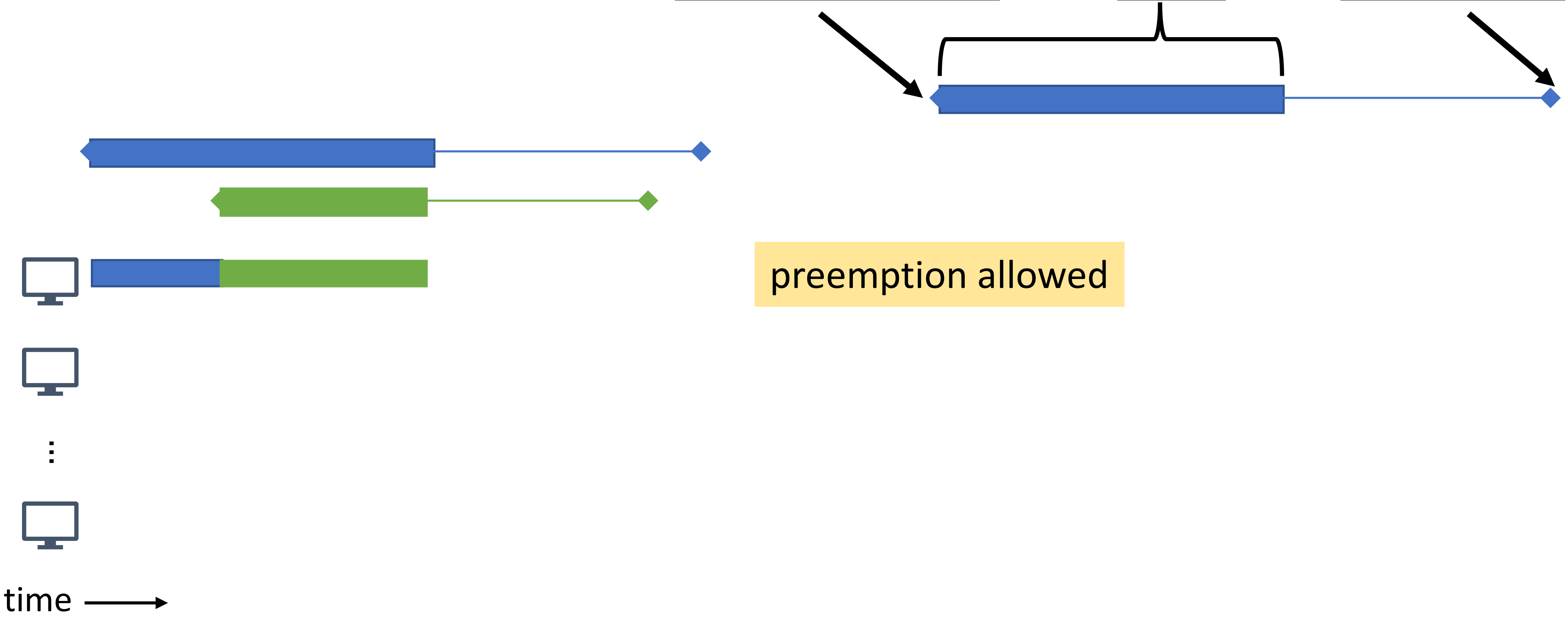

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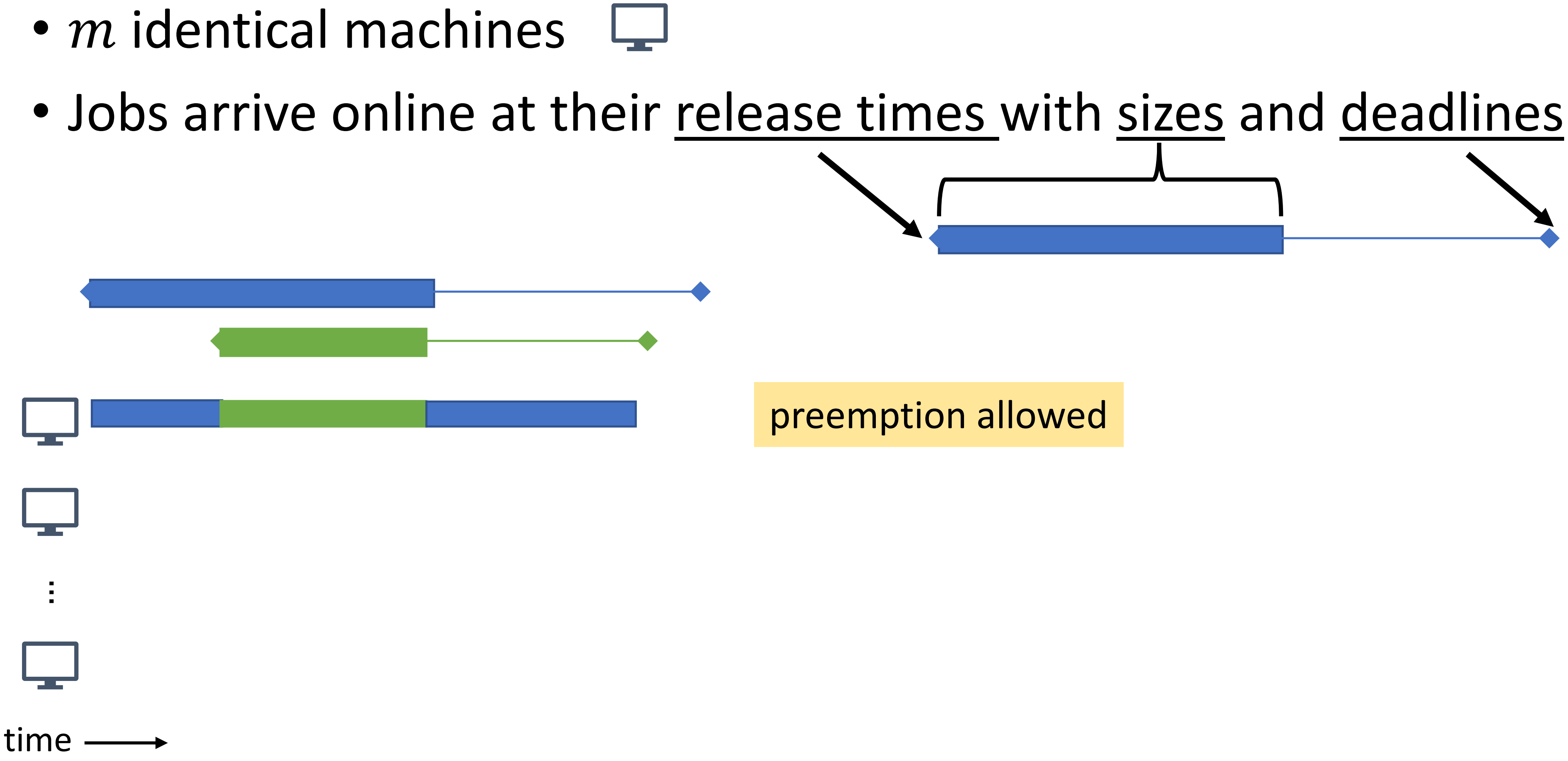

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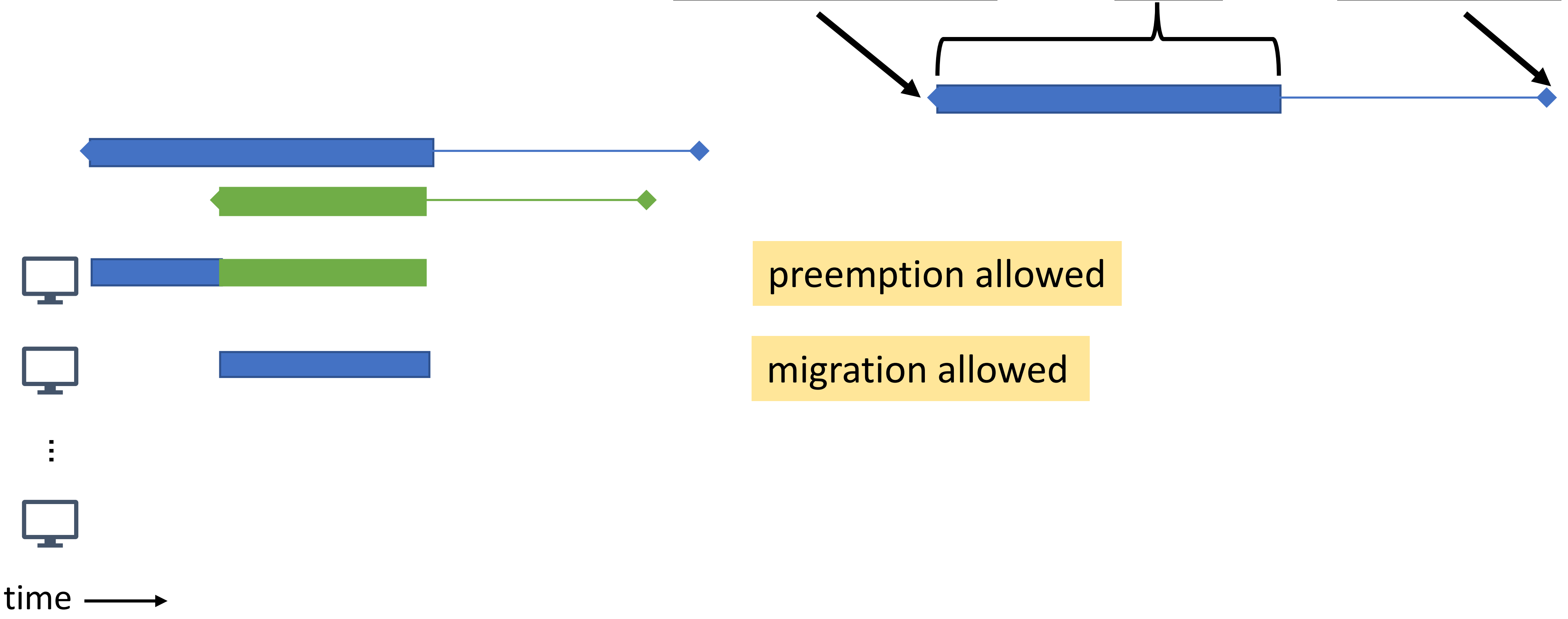

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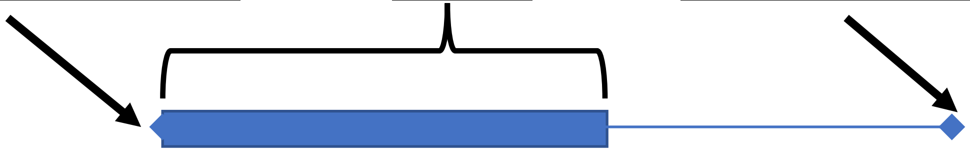

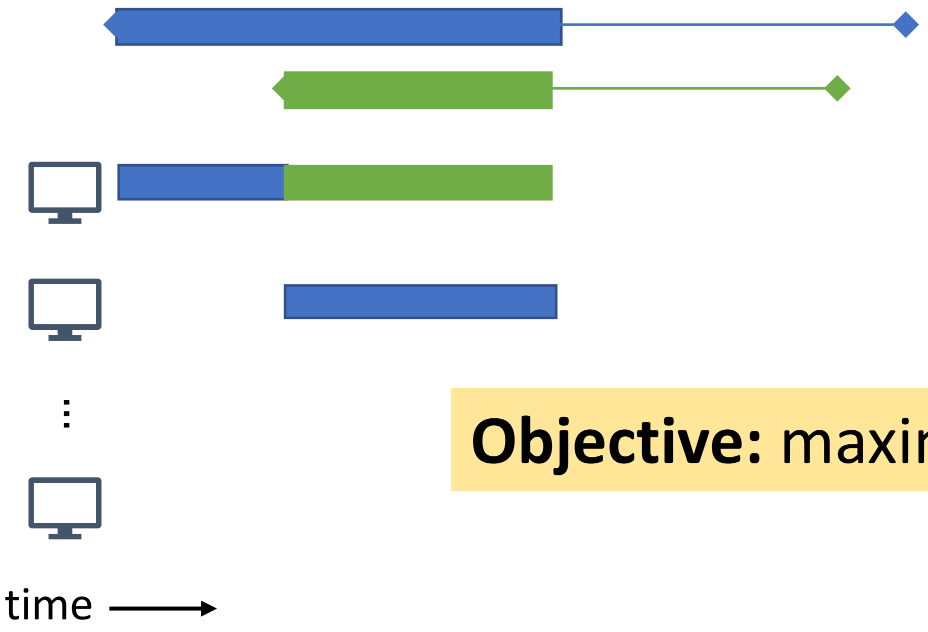

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migration allowed

Objective: maximize # jobs completed by deadline

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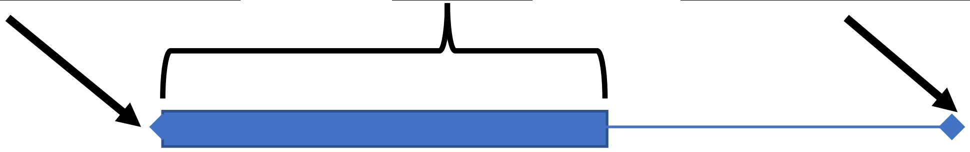

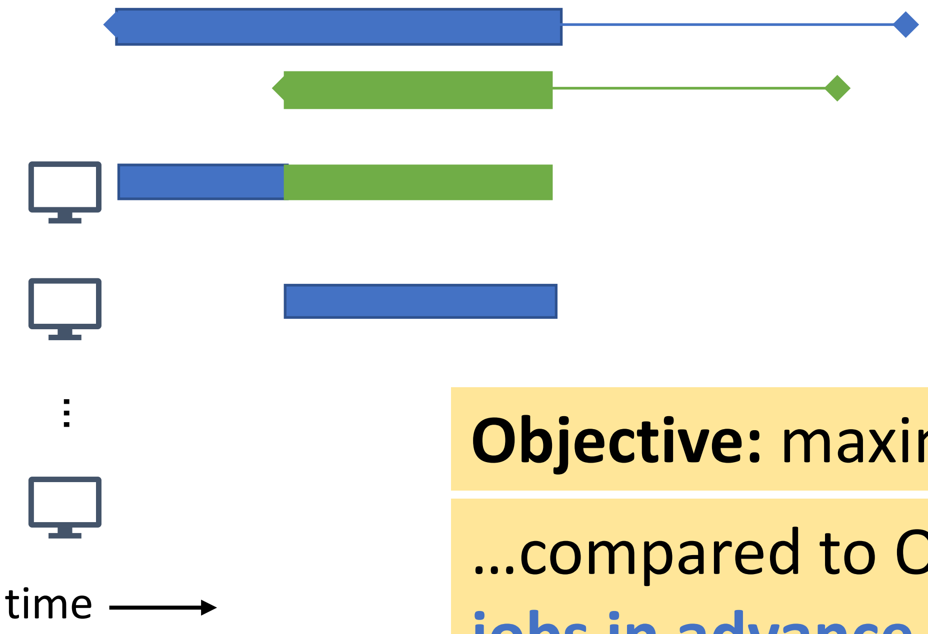

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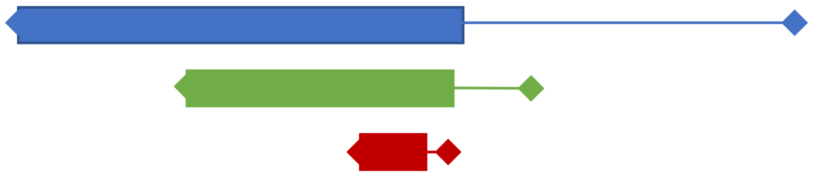
…compared to Opt. offline sched. that **knows all jobs in advance**

Hard Example



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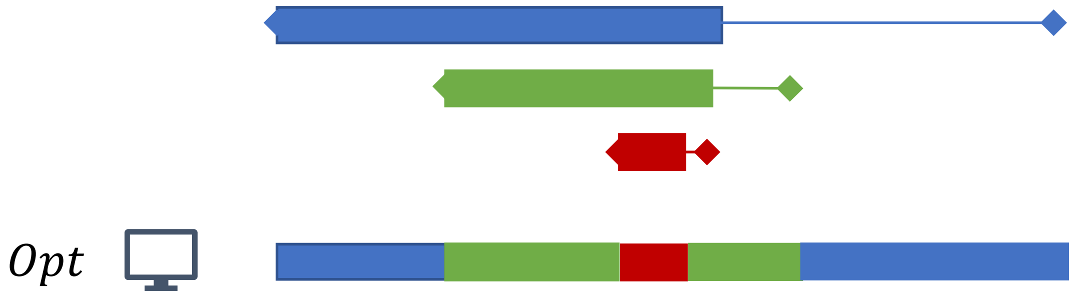

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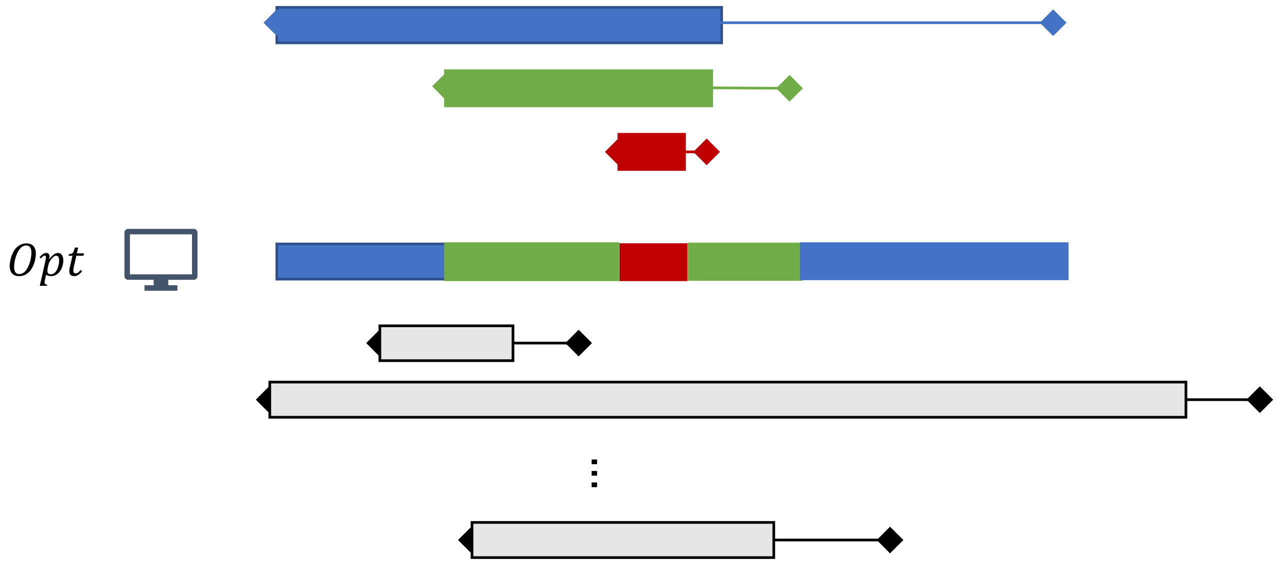

Q: How can Alg. know what is the 'right' job to preempt for?

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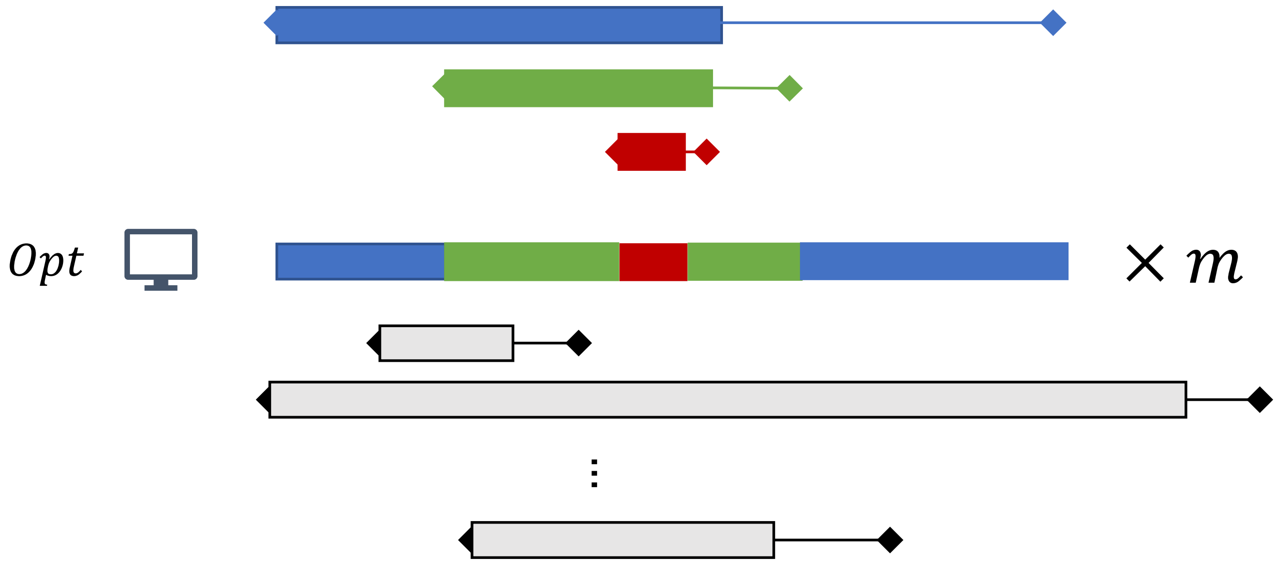

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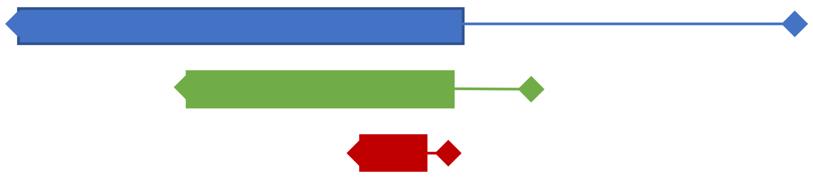
Laxity



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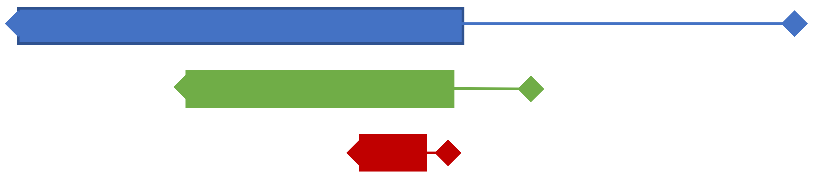

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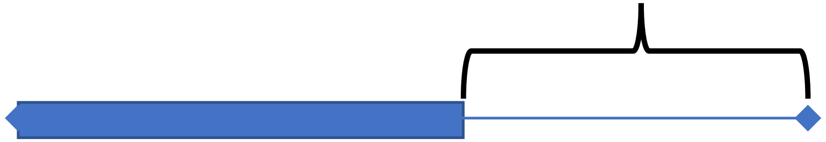

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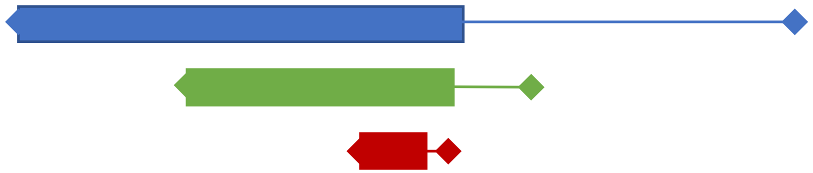

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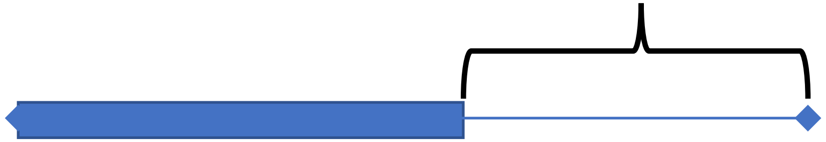

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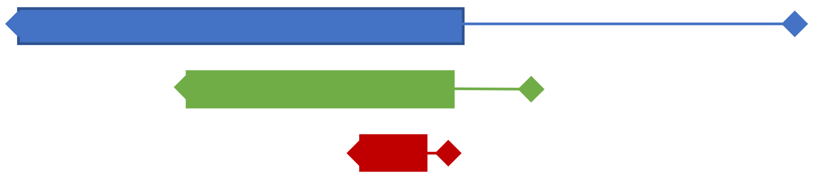

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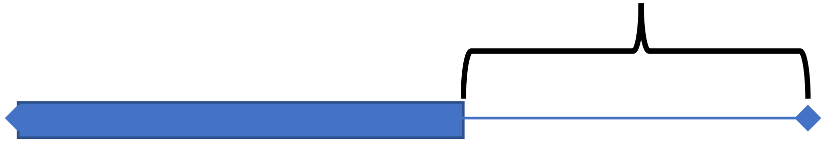

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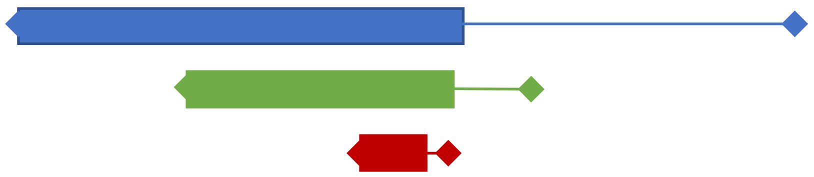

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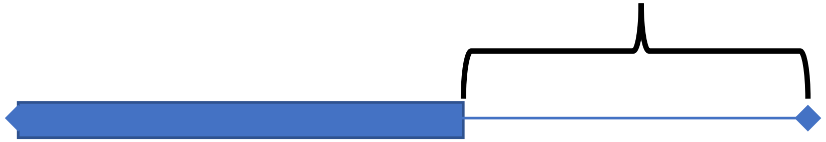

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Main Challenge: How to handle jobs where $\boldsymbol{laxity \ll size}$?

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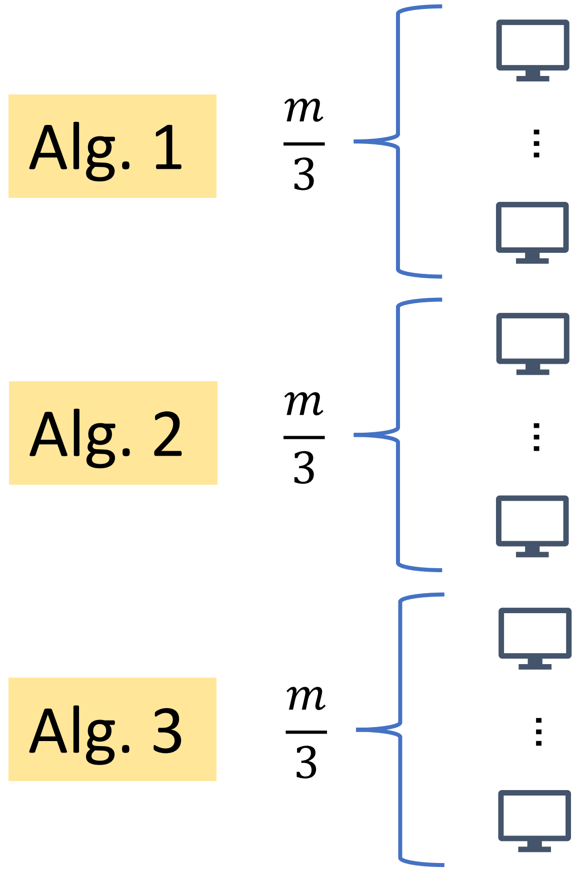

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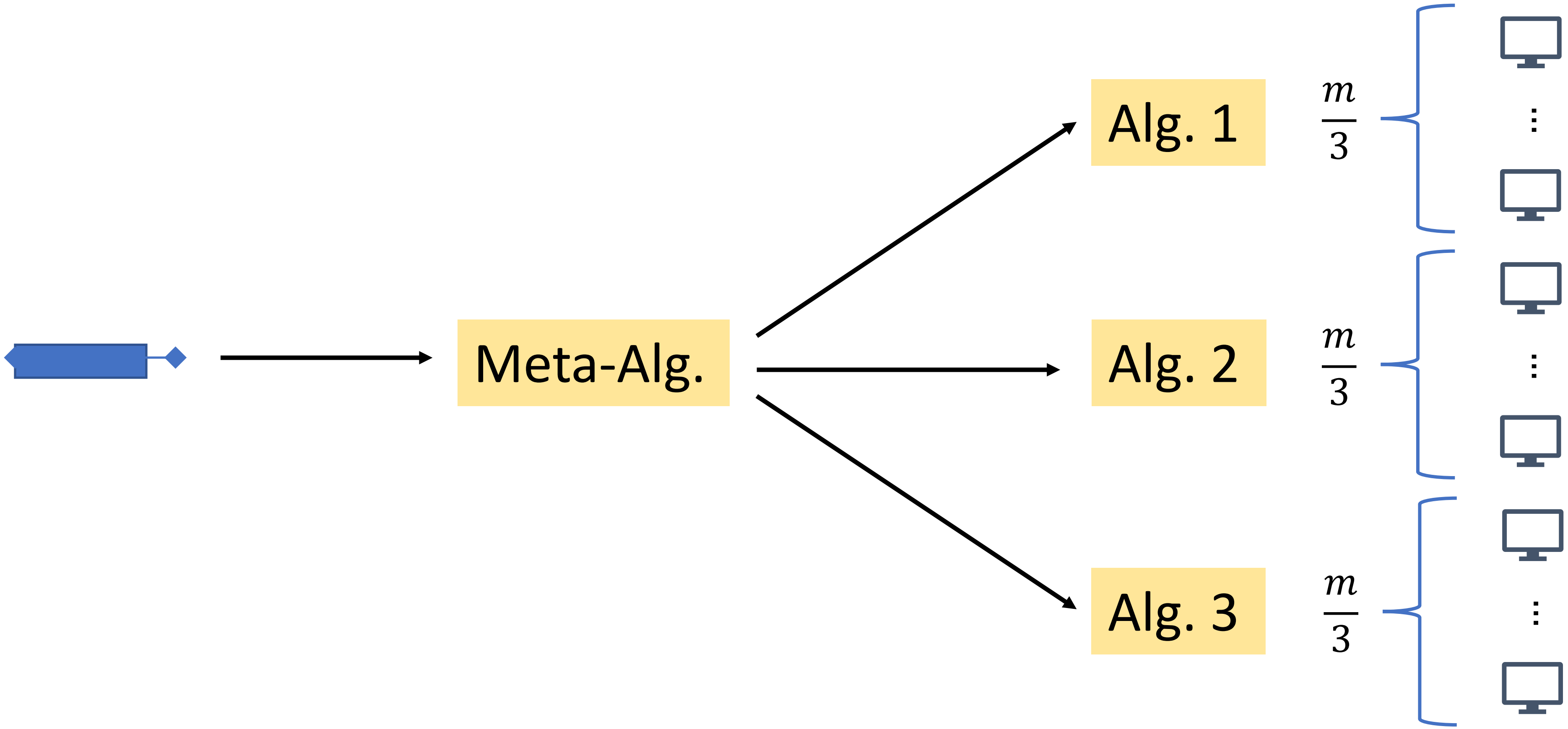

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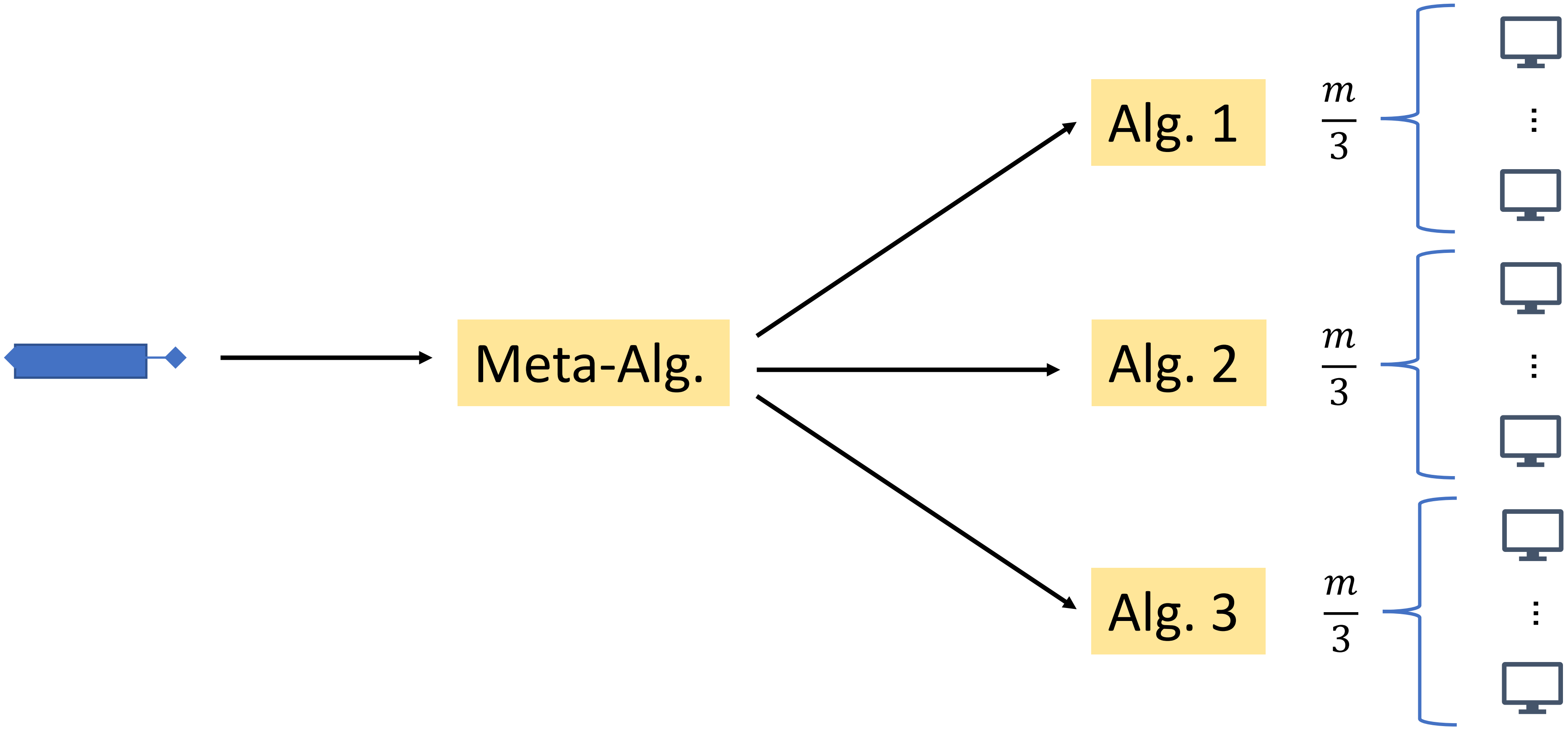

Idea: Meta-Alg. distributes jobs to 3 Algs.
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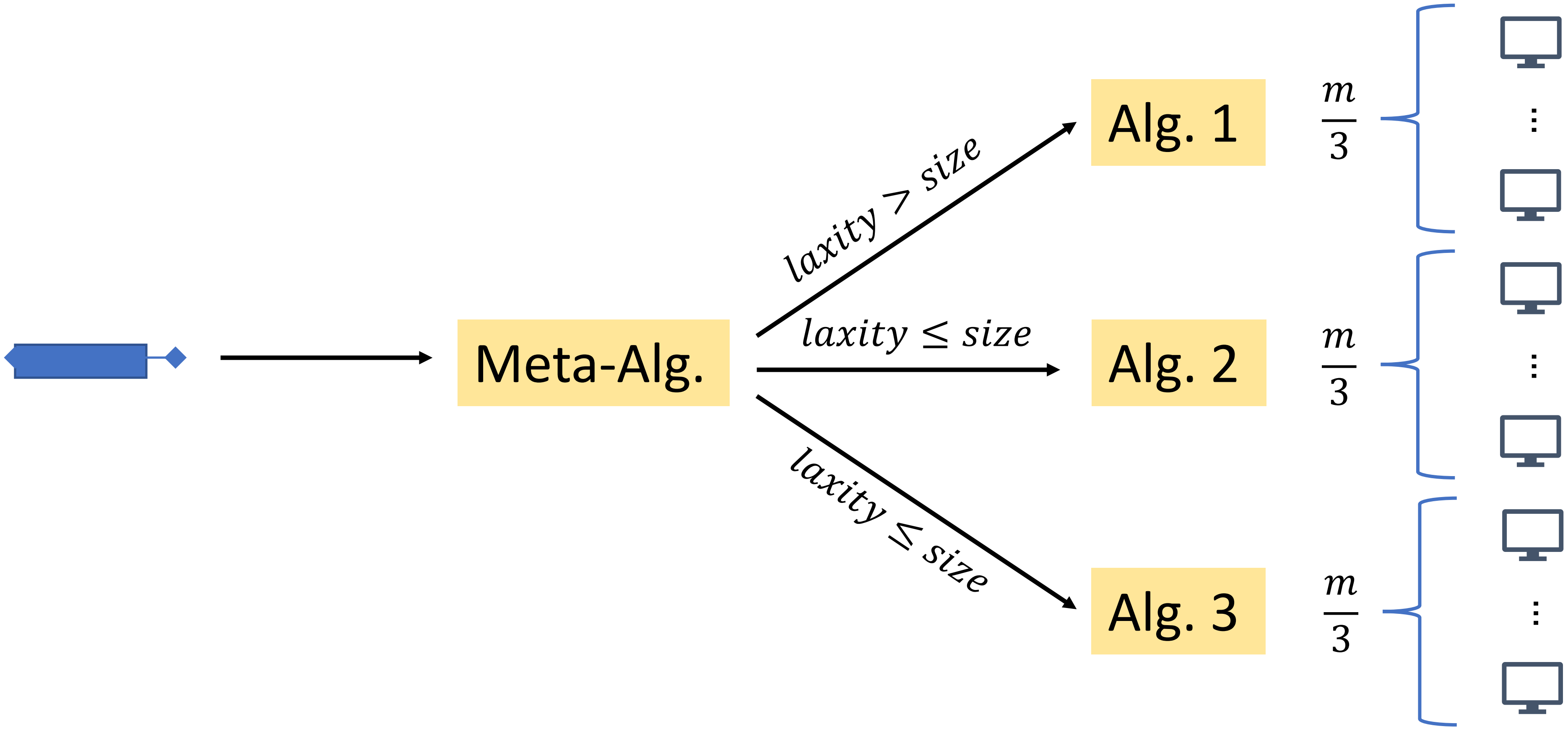

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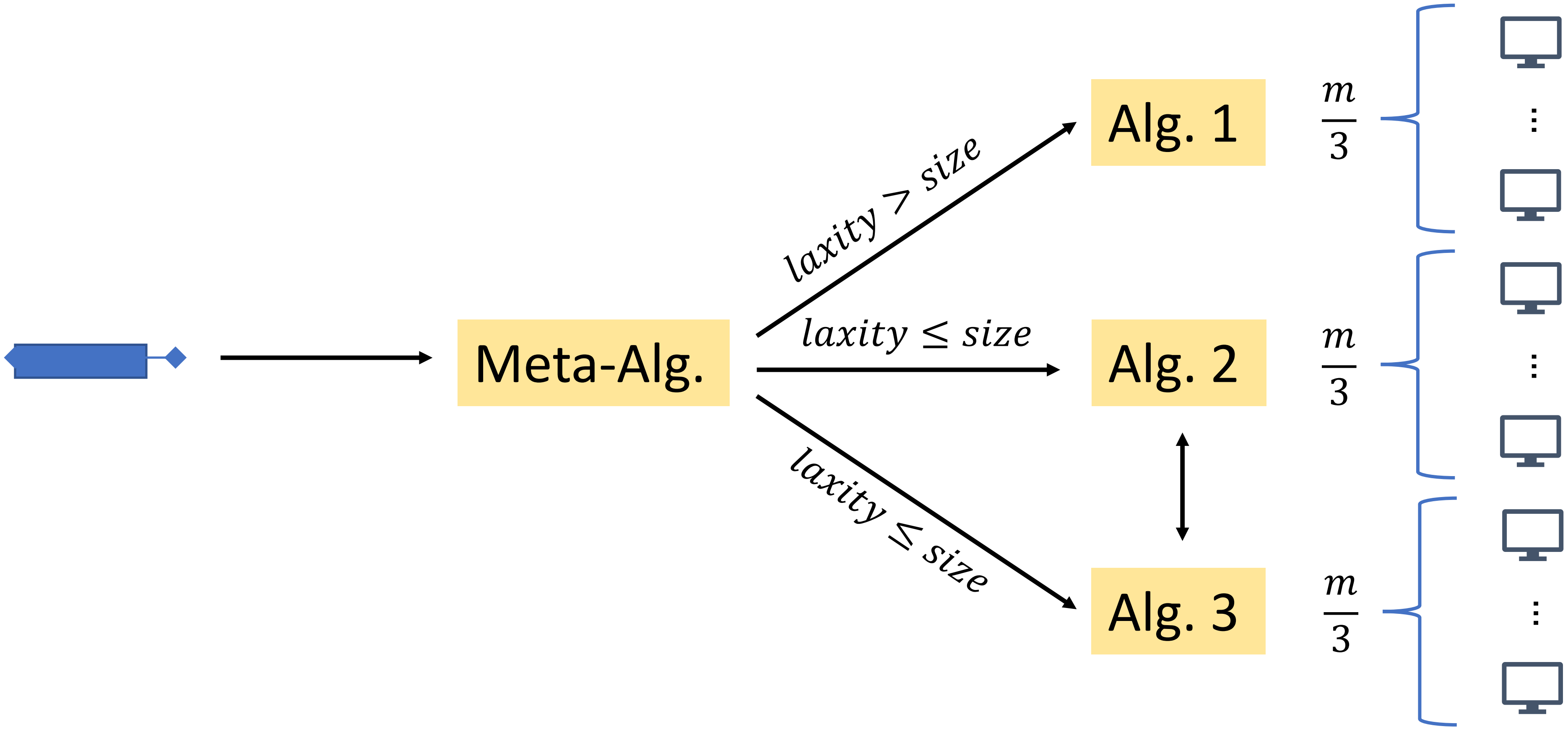

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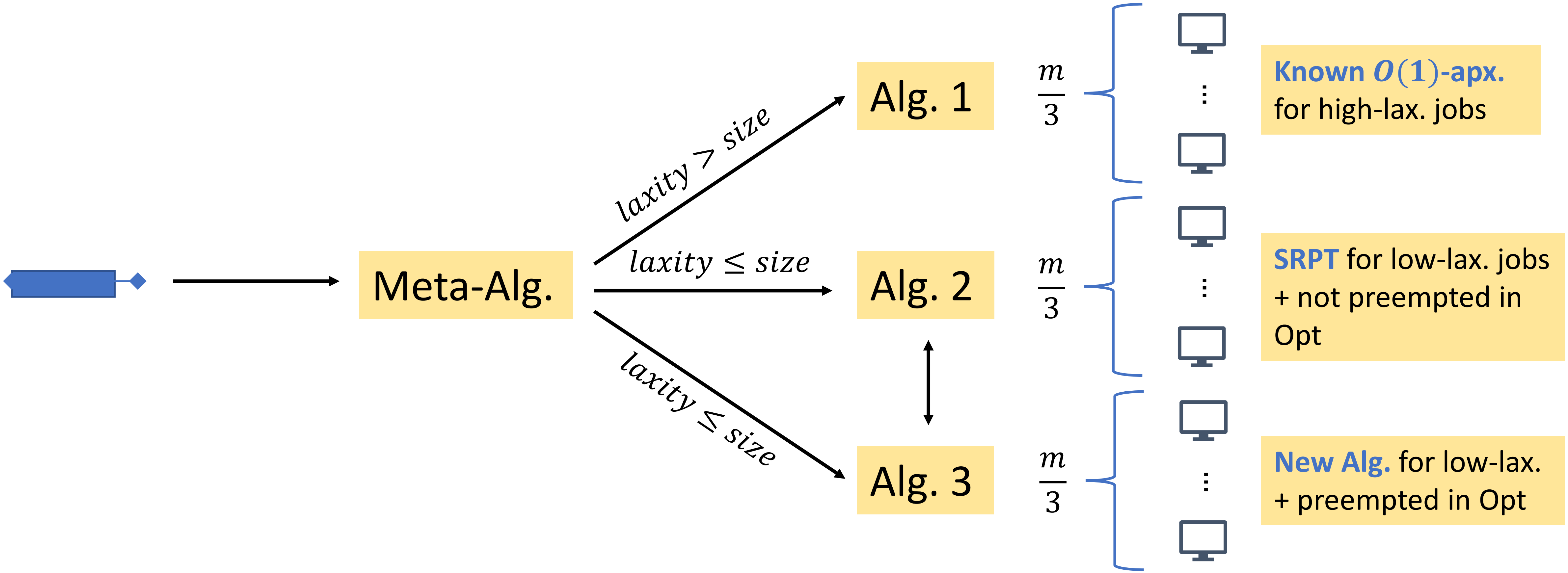

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Low-Laxity Jobs

Opt

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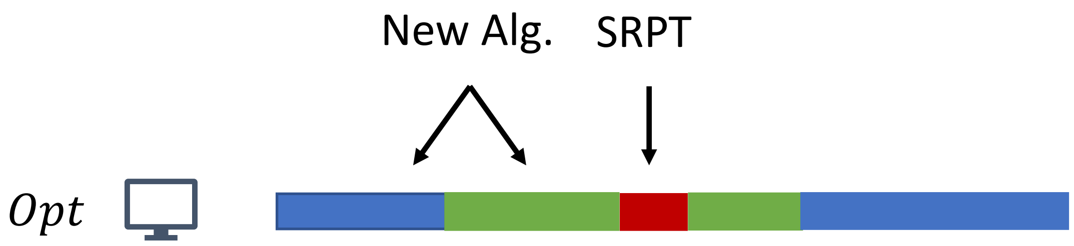

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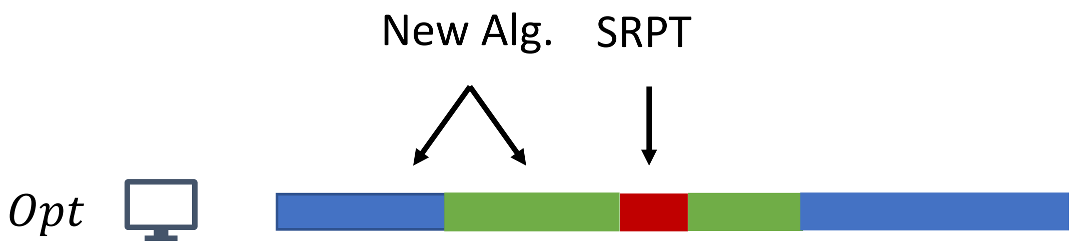

- **SRPT Sketch:**
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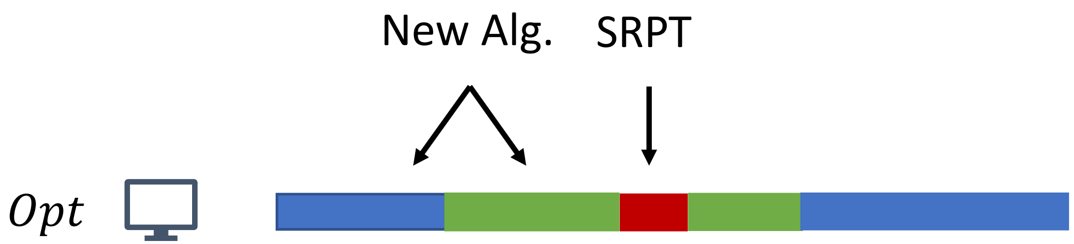
  - Else when Opt runs red, SRPT must be running m jobs with even shorter remaining size than red ⇒ ☺

Low-Laxity Jobs



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Main Challenge: How does **New Alg.** handle jobs where $\boldsymbol{laxity \le size}$ **+ preempted by Opt?**

Alg. for Low-Laxity + Preempted

- **Idea:** each mach. maintains a stack of jobs $\sim$ chain of preemptions

$Alg(\times m)$

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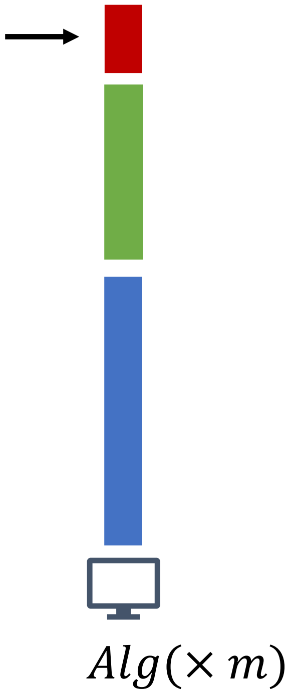

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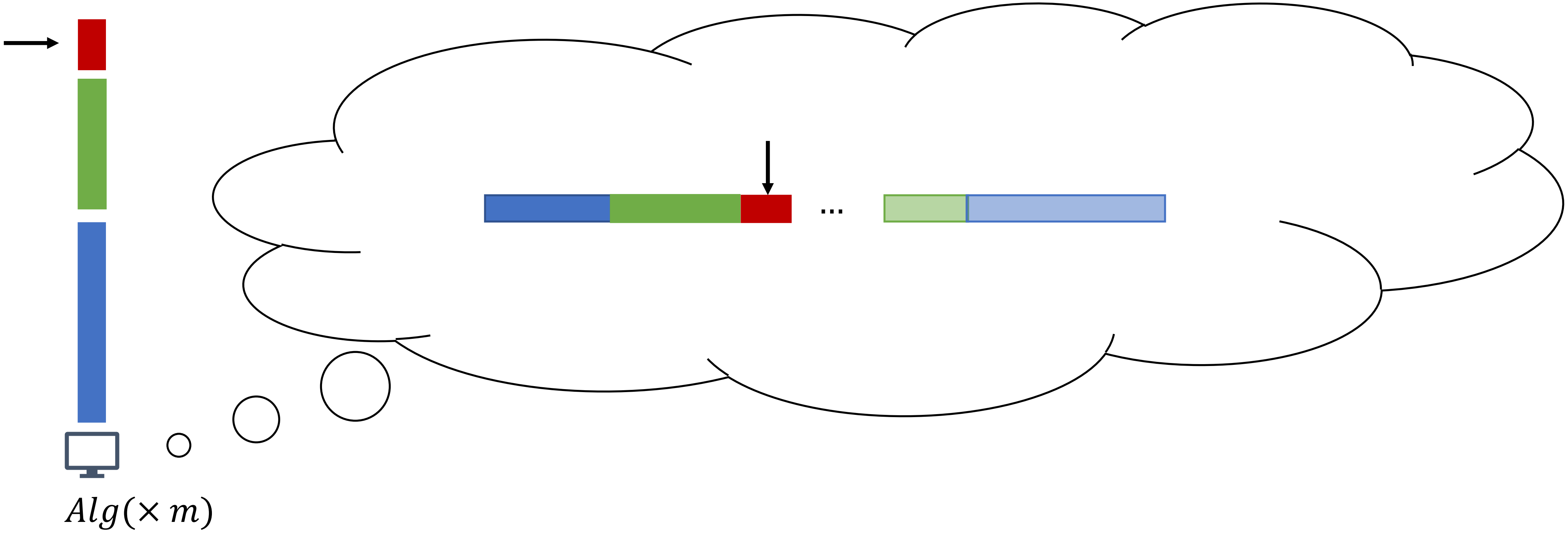

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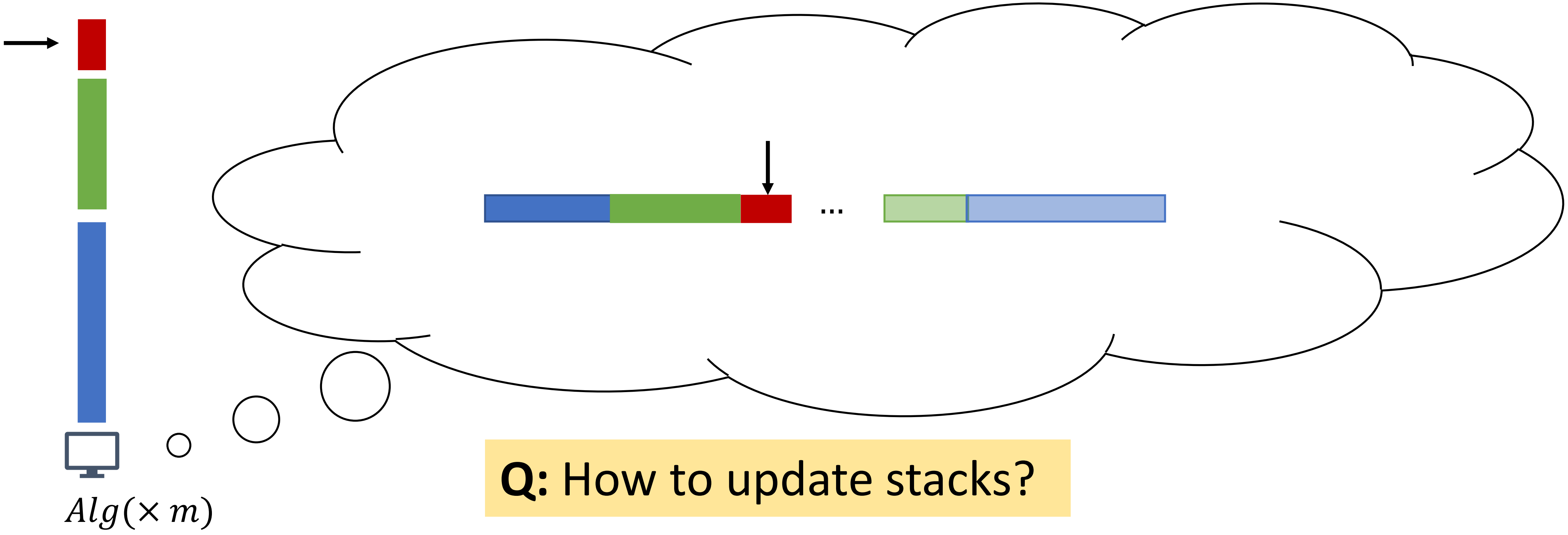

Q: How to update stacks?

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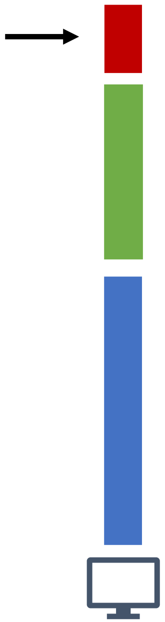

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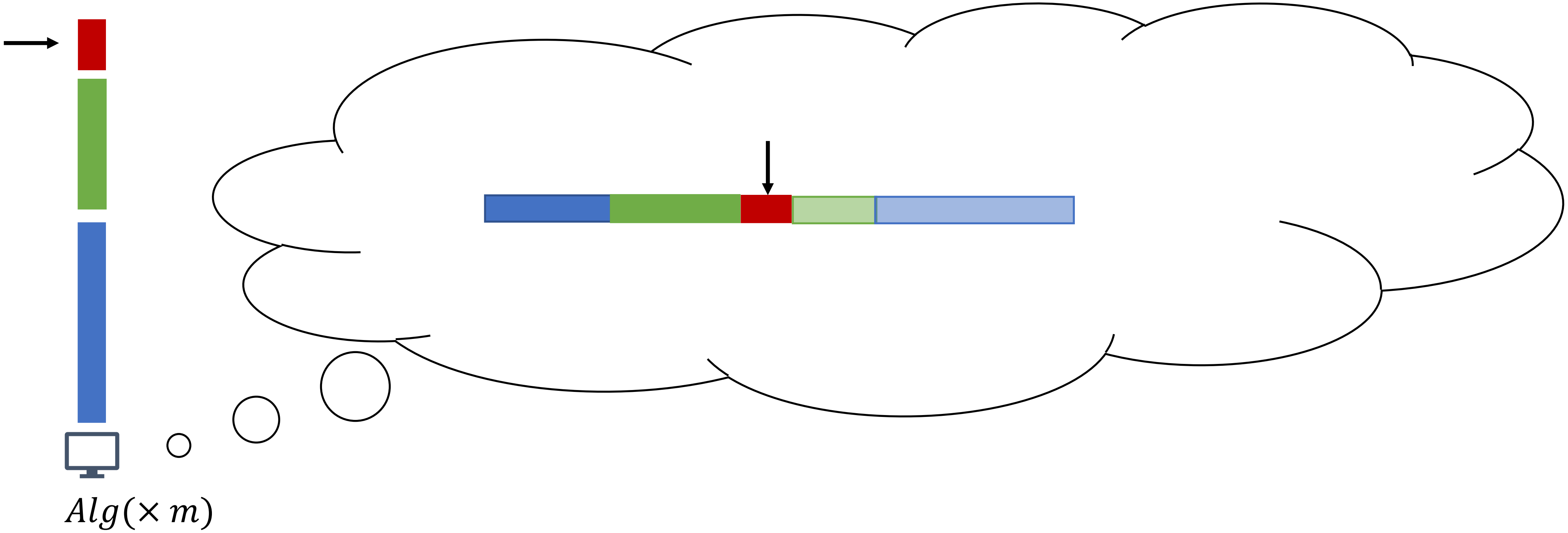

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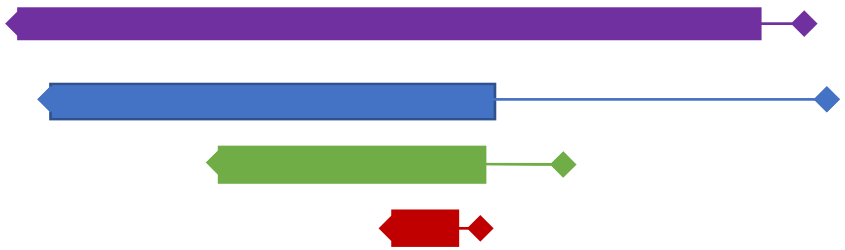

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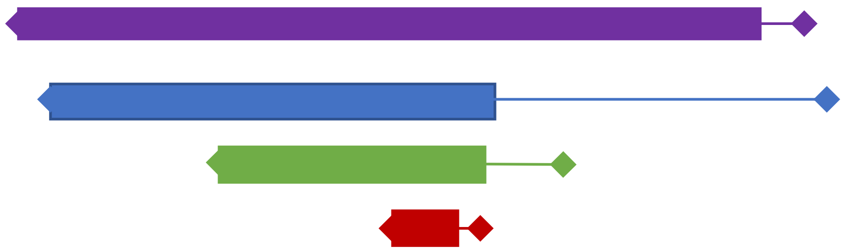

Need to sometimes replace current job

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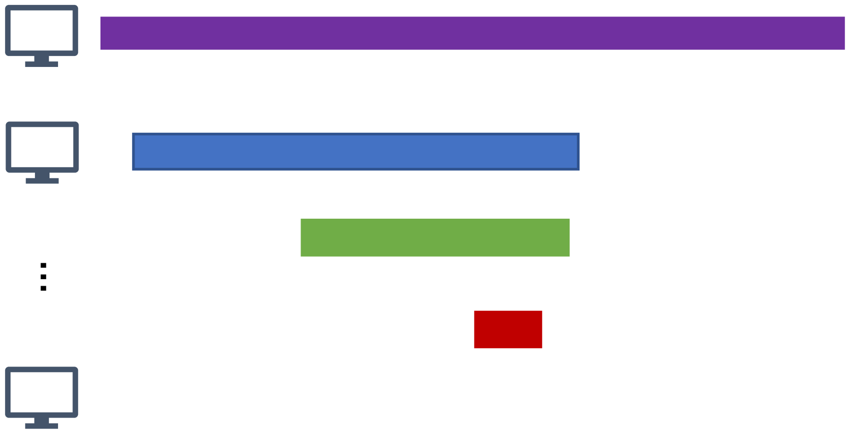
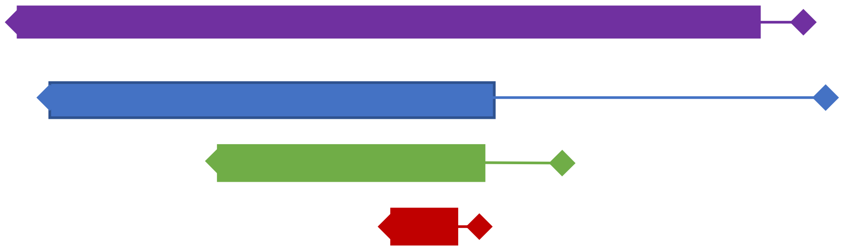
Final Alg. – Replacements

- Recall: m machines





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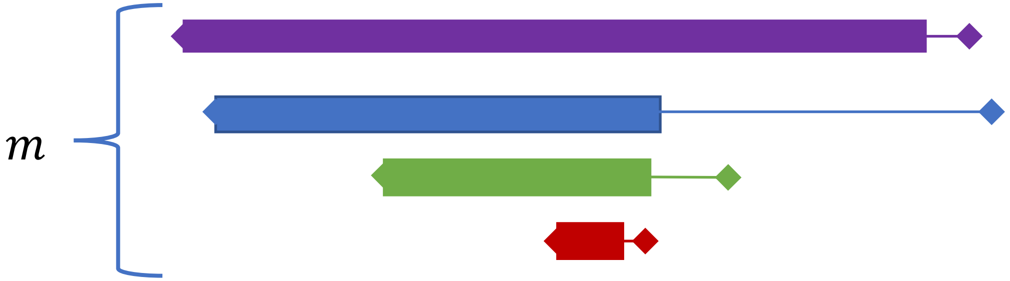

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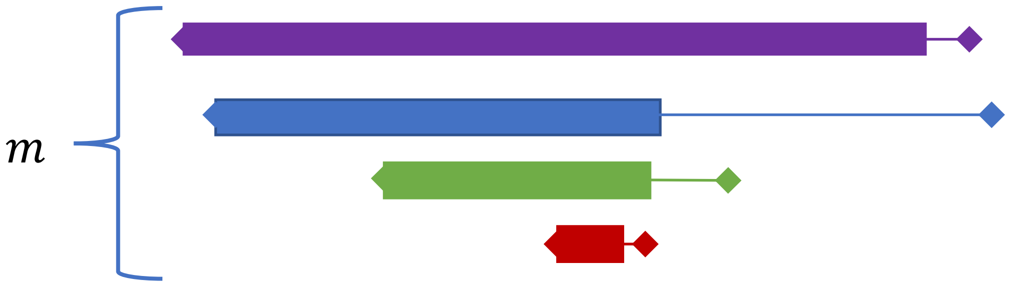

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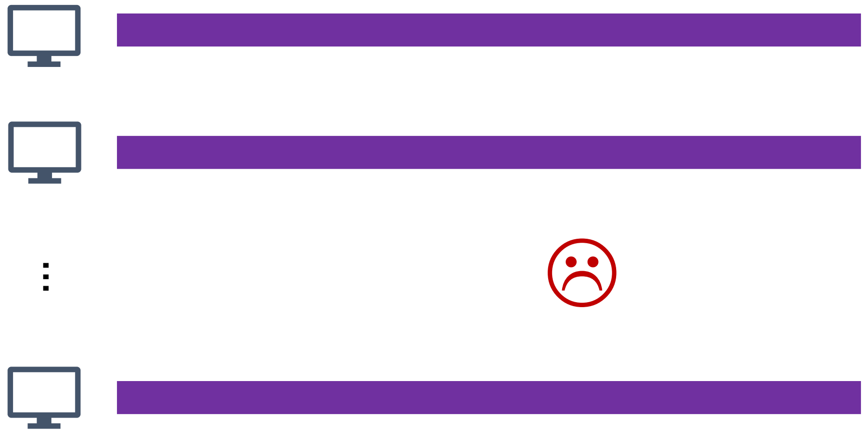

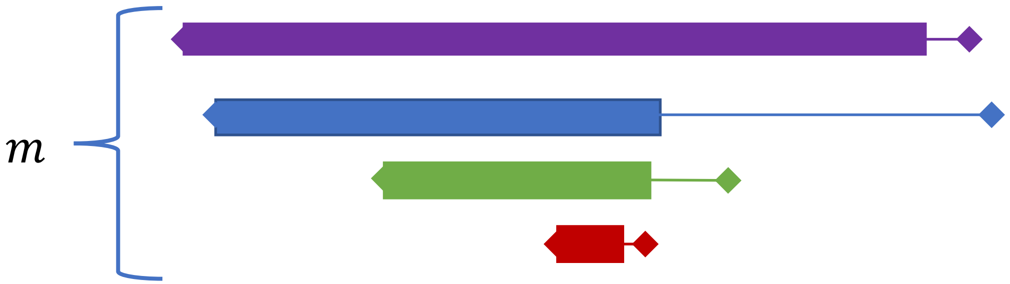

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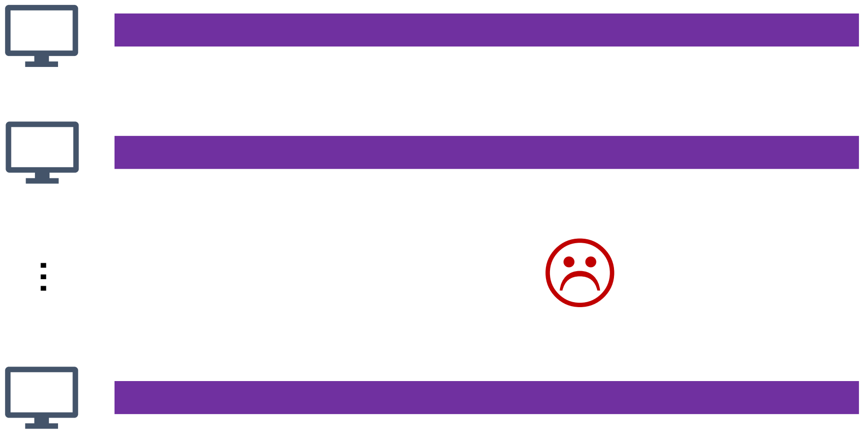

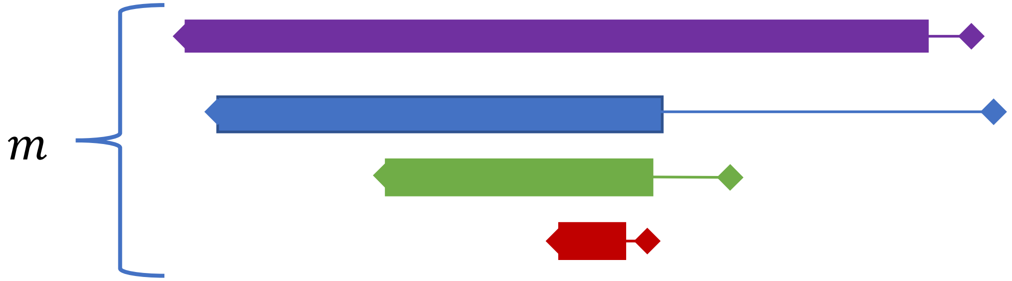

Idea: If new job is much better than majority of the stacks, then replace

Final Alg. – Replacements

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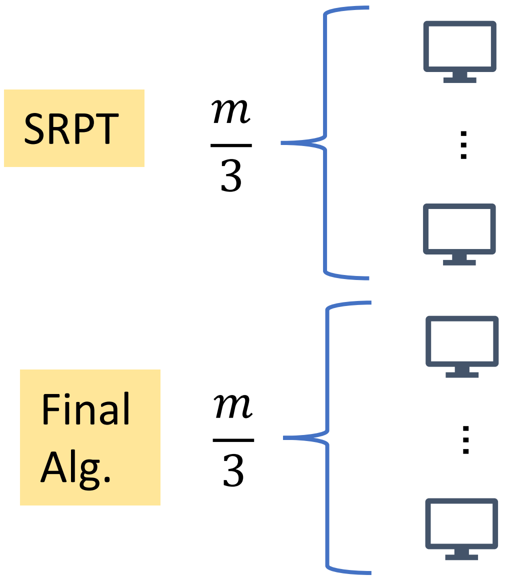
*some tie-breaking rules apply

Analysis Sketch

- Show # $(Pushes\ by\ Final\ Alg.) = \Omega(\#(Completions\ by\ Opt))$



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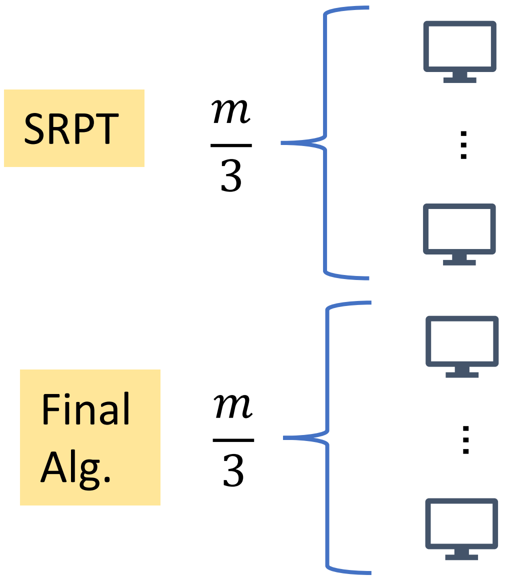

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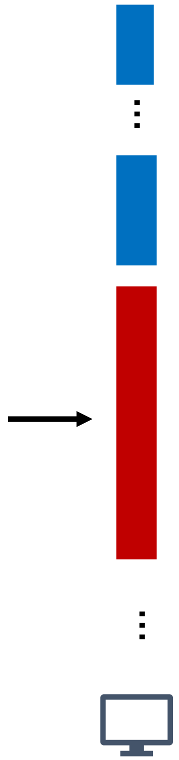
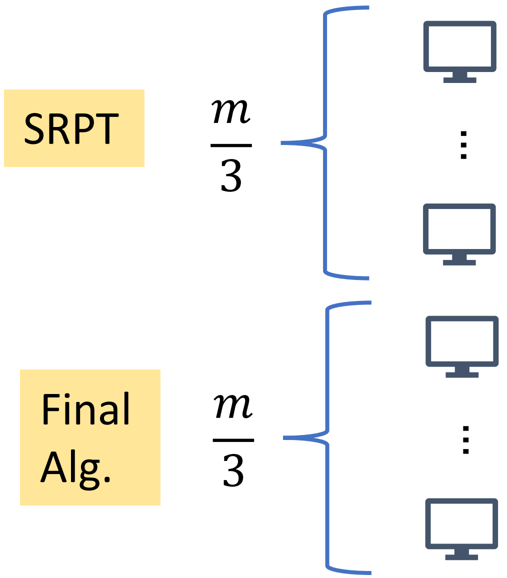
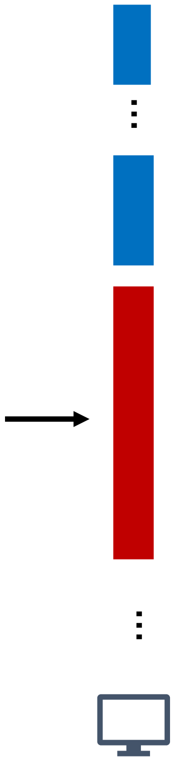
  - If job j is popped due to being infeasible:
    - $\Rightarrow$ many pushes/replacements on j's stack

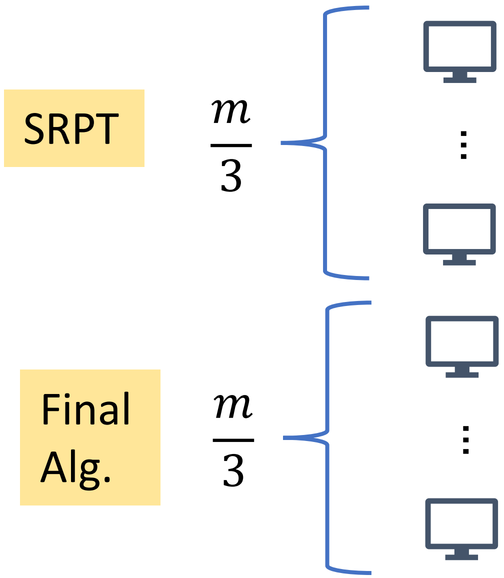




Analysis Sketch

- Show # $(Pushes\ by\ Final\ Alg.) = \Omega(\#(Completions\ by\ Opt))$
- Show do not pop many jobs due to being infeasible
  - If job j is popped due to being infeasible:
    - $\Rightarrow$ many pushes/replacements on j's stack
      - If many pushes, then charge to completion of those pushes
      - If many replacements, then can find long interval where Final Alg. is running jobs much smaller than $j \Rightarrow$ witness that SRPT is running even better jobs





Summary

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- Concludes 20-year line of research since $m = 1$ case settled
- **Algorithm:** Run 3 algs. on $m/3$ machines each

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Open Question: Can make Alg. non-migratory?