Online Demand Scheduling with Failovers

Konstantina Mellou Marco Molinaro **Rudy Zhou***

Microsoft Research, Redmond Carnegie Mellon

* work performed as intern in Cloud Operations Research (CORE) group at Microsoft Research, Redmond

Robust Assignments

- What happens if a machine fails?

Robust Assignments

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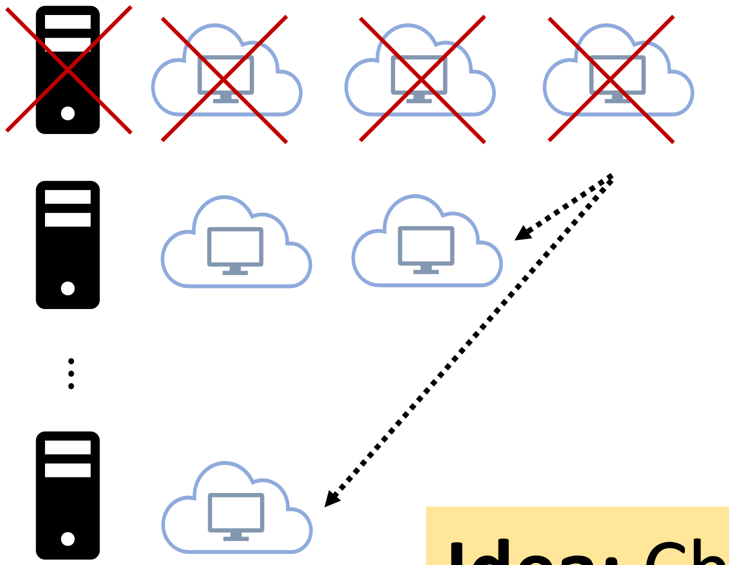
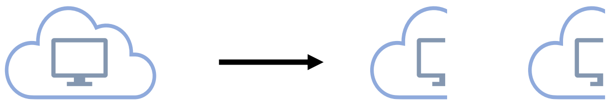
Robust Assignments

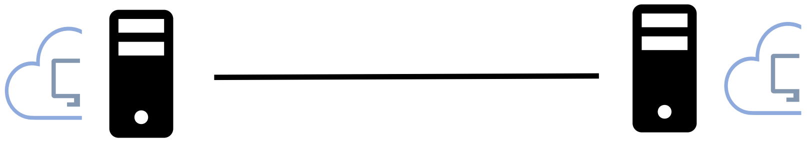
- What happens if a machine fails?
- How to reassign?



Idea: Change assignment process to make reassignment in case of failure easier

New Model: Redundancy

- Inspired by real systems architectures
- Split each demand in half: 
- Assign each half to distinct machines



Chaojie Zhang, Alok Gautam Kumbhare, Ioannis Manousakis, Deli Zhang, Pulkit A. Misra, Rod Assis, Kyle Woolcock, Nithish Mahalingam, Brijesh Warrier, David Gauthier, Lalu Kunnath, Steve Solomon, Osvaldo Morales, Marcus Fontoura, Ricardo Bianchini:
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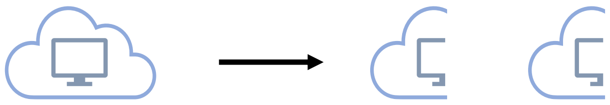
- Split each demand in half: 
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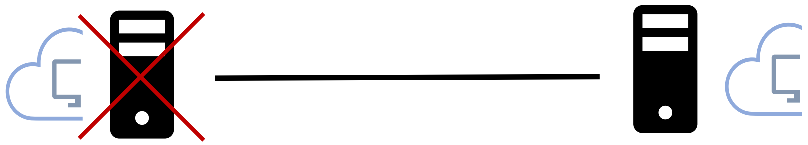

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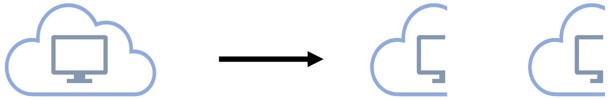
- Split each demand in half: 
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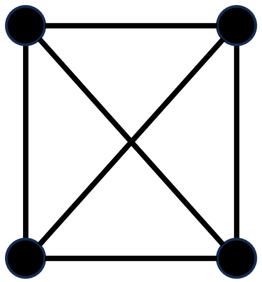
Online Demand Scheduling with Failover

- m machines



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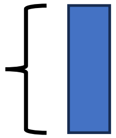
- n demands arrive online with sizes: 
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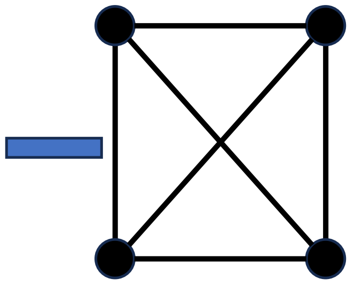

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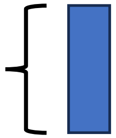
- n demands arrive online with sizes: 
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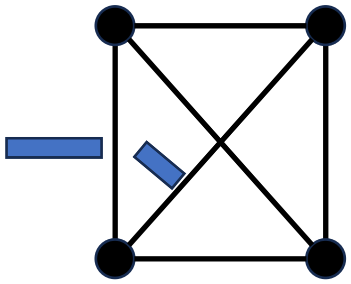

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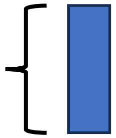
- n demands arrive online with sizes: 
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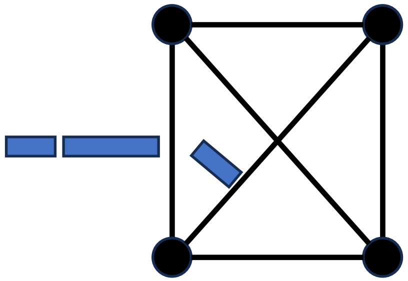

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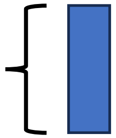
- n demands arrive online with sizes: 
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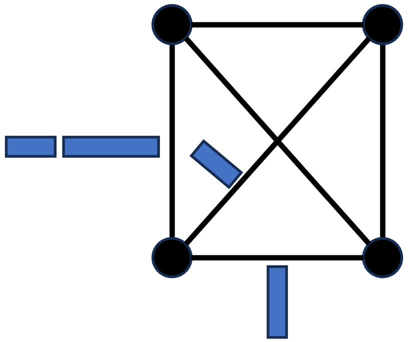

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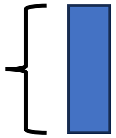
- n demands arrive online with sizes: 
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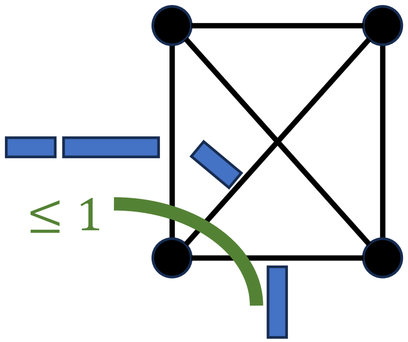

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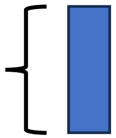
- n demands arrive online with sizes: 
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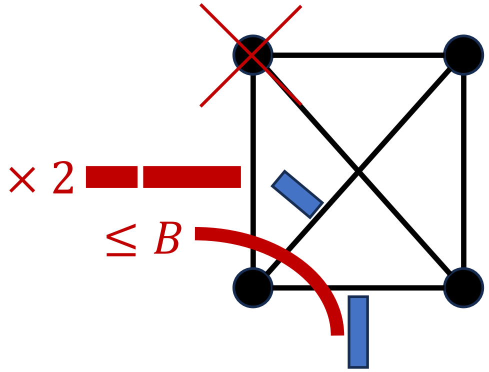
for every machine u



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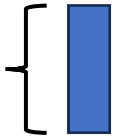
- n demands arrive online with sizes: 
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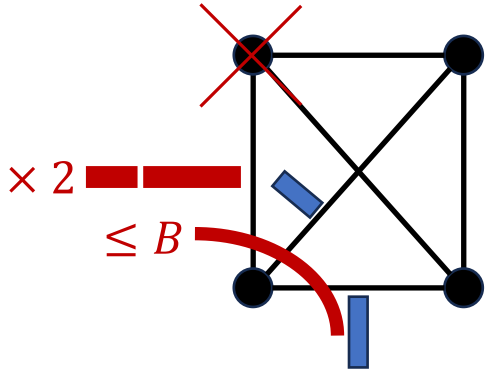

Goal: maximize utilization (total size of assigned demands) until first demand that needs to be rejected

Related Work

- Multiple Knapsack [Chandra Chekuri, Sanjeev Khanna. SIAM J. Comput. 2005]
- Coupled placement [Madhukar R. Korupolu, Adam Meyerson, Rajmohan Rajaraman, Brian Tagiku. Math. Prog. 2015]
- Do not capture failover constraints (depends on how load is arranged on a machine's edges)

Our Results

Compared to optimal offline policy that knows all demands but also assigns demands in same order until rejecting

Theorem (Worst Case): $(\frac{\mathbf{1}}{\mathbf{2}} - o(1))$ - competitive deterministic algorithm

Theorem (Stochastic): If all demand sizes are drawn i.i.d. from an unknown distribution, then $(\mathbf{1} - o(1))$ – competitive algorithm w.h.p.

- No deterministic algorithm is better than $\frac{1}{2}$ - competitive

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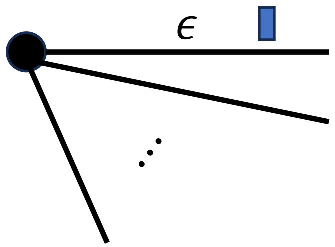

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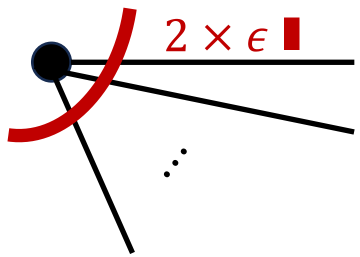

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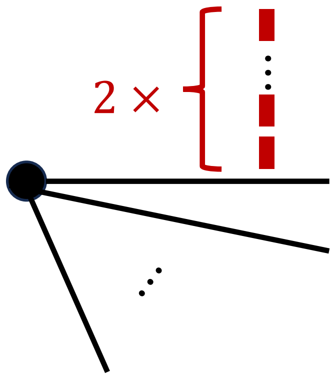

- Minimize impact of failover by spreading out demands

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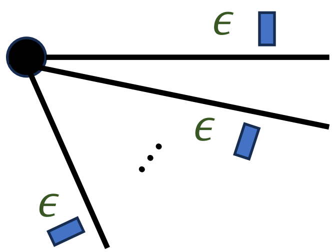

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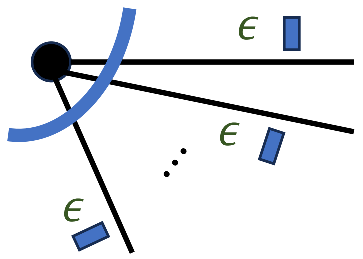
$degree = \frac{1}{\epsilon} - 1$ (assume $B = 1$)



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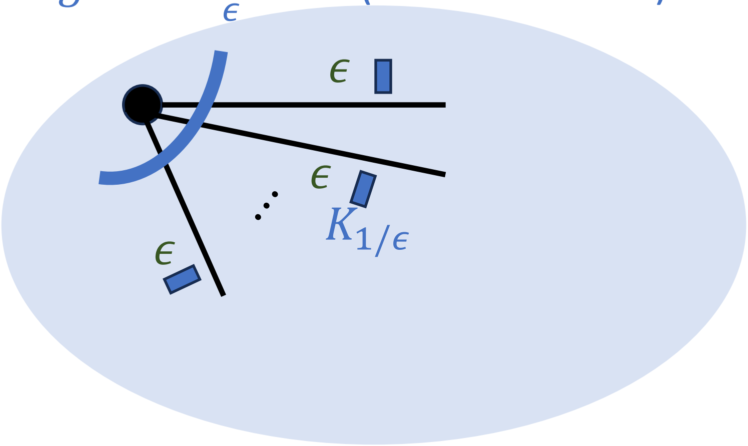

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- Ideally, want to make clique of machines for same size demands

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- Ideally, want to make clique of machines for same size demands
- … but need to make sure we don't run out of machines

Worst Case: Algorithm

- Assume $B = 1 \Rightarrow$ want to arrange demands of size $\frac{1}{k}$ in a K_k
- Assume all demand sizes are $\frac{1}{k}$ for integer k

Algorithm: If a demand of size $\frac{1}{k}$ arrives, assign it to an open edge in a reserved K_k; otherwise reserve a new K_k and assign it there

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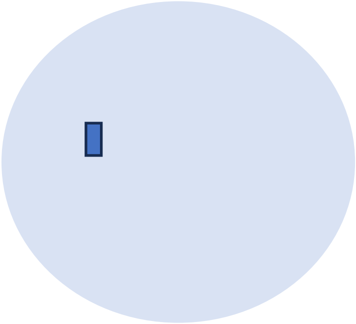

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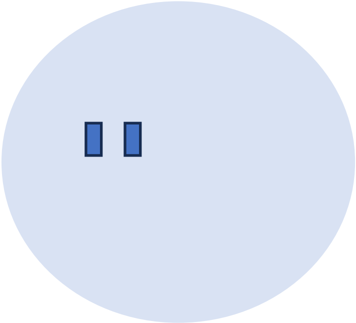

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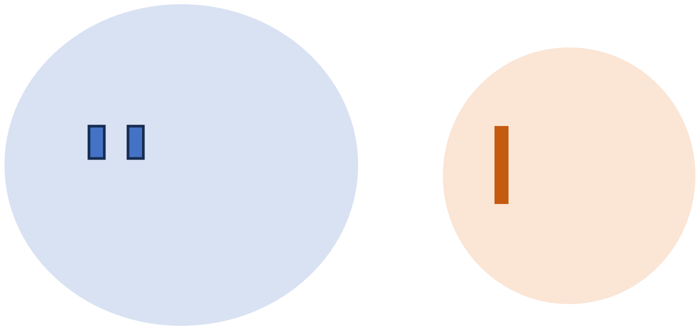

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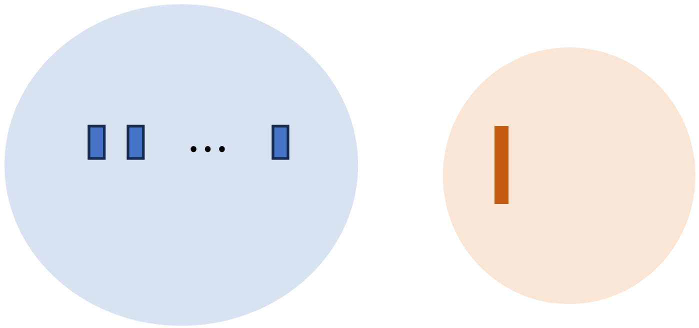

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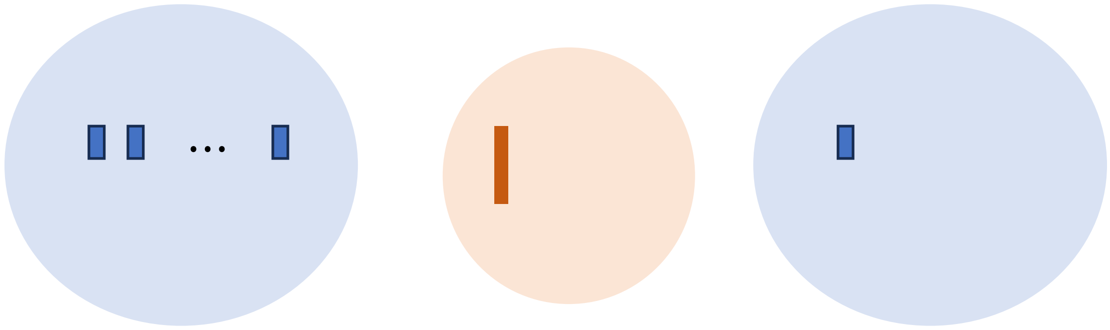

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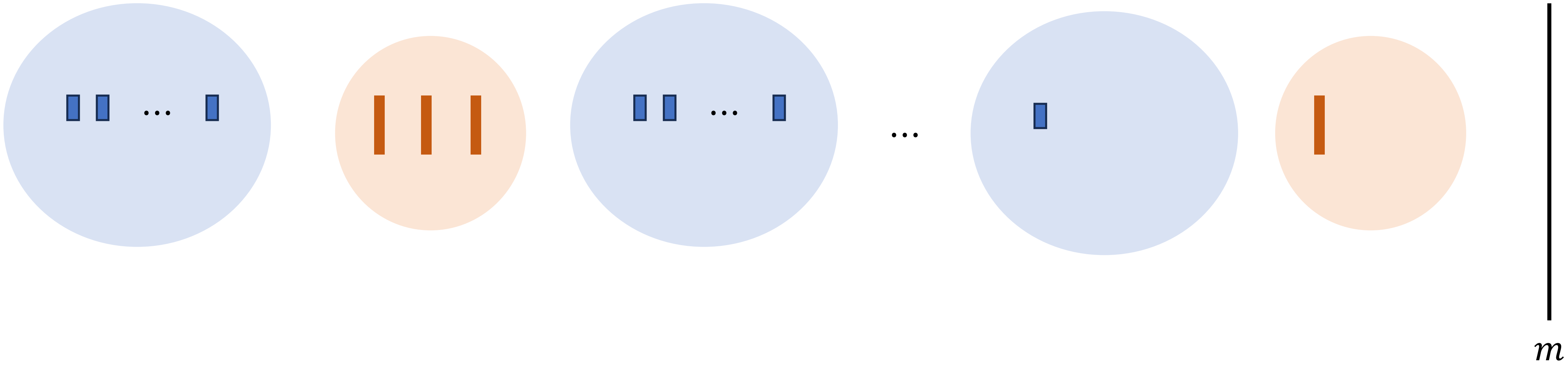

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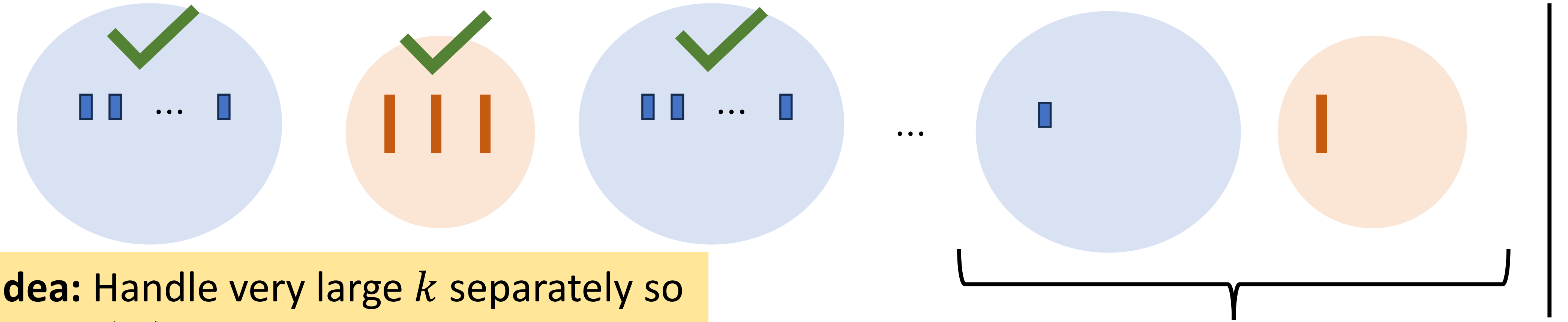

Idea: Handle very large k separately so $k \leq o(m) \Rightarrow$ waste $o(m)$ machines

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Theorem (Stochastic): If all demand sizes are drawn i.i.d. from an unknown distribution, then $(1 - o(1))$ – competitive algorithm w.h.p.

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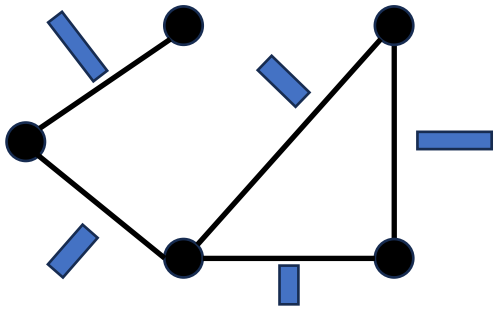
Stochastic Model: Idea

- Each demand size drawn i.i.d. from distribution μ

Algorithm: Suppose we already assigned first n' demands:
- Compute (near-) optimal assignment of realized demand sizes into minimum number of machines
- Use this assignment to assign the subsequent n' online arrivals

Realized first n'



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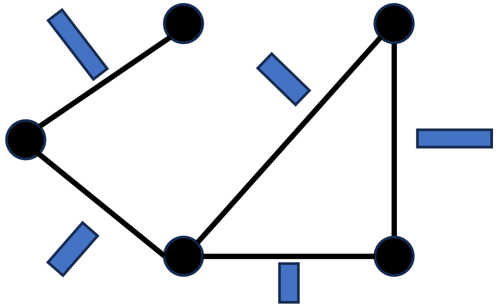
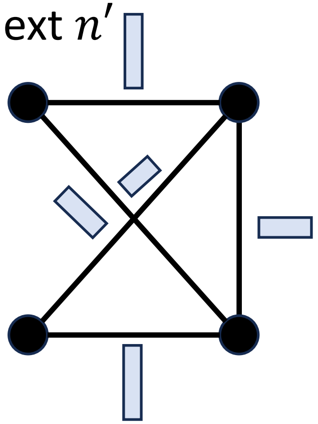


Template assignment for next n'



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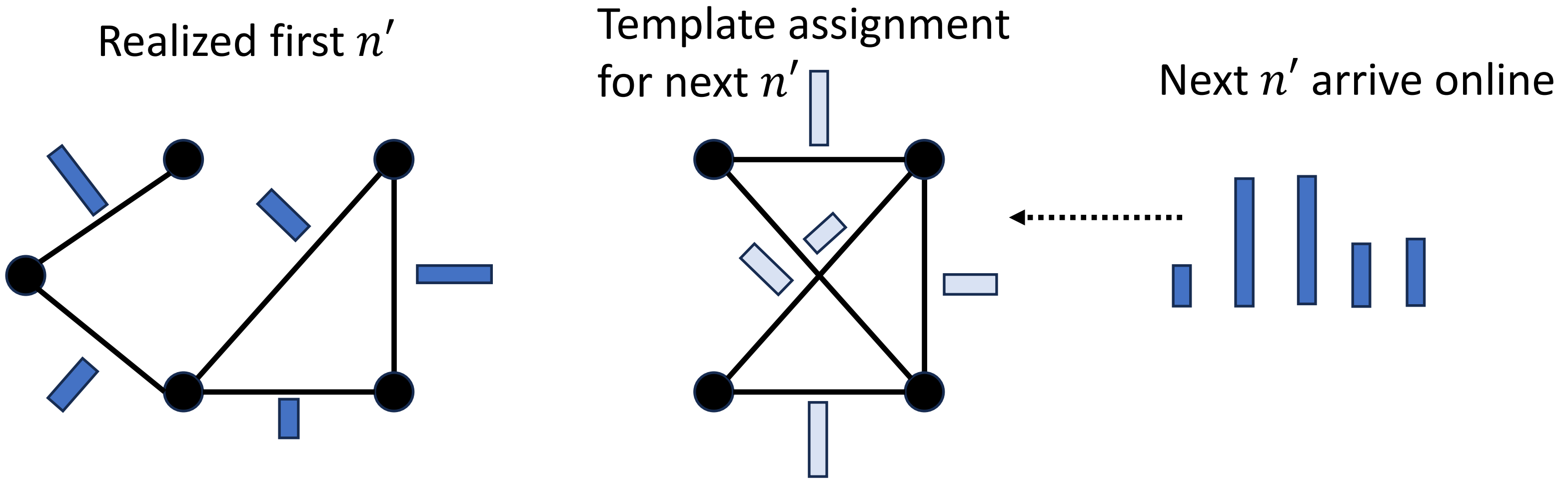

Template Assignment

Theorem (Monotone Matching): Given sequences $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ (arriving online) drawn i.i.d. from the same distribution, we can compute a matching from Y's to X's such that w.h.p.:

- If Y_i is matched to X_j, then $Y_i \leq X_j$
- At most $o(n)$ of the Y's are unmatched

How to use template:

- Compute monotone matching from n' next online arrivals to the realized first n'
- If matched, then assign arrival to corresponding slot in template
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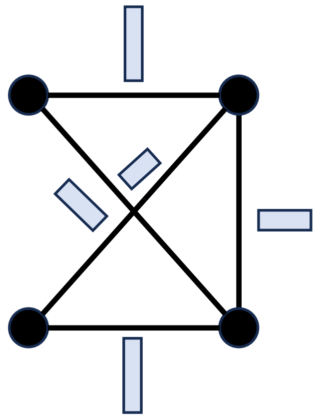

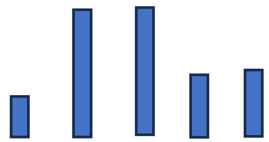

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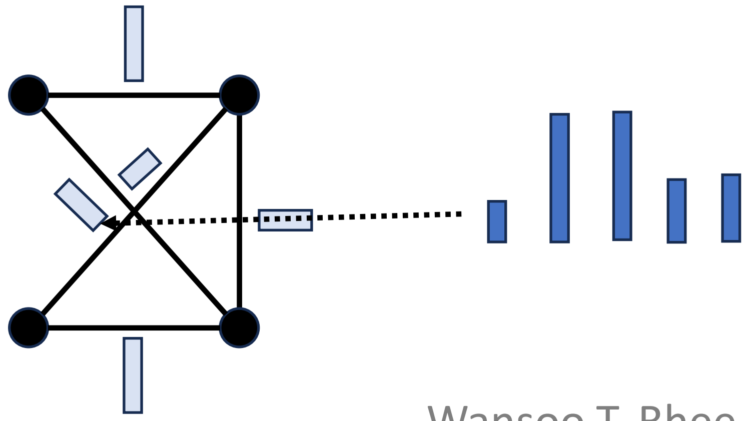

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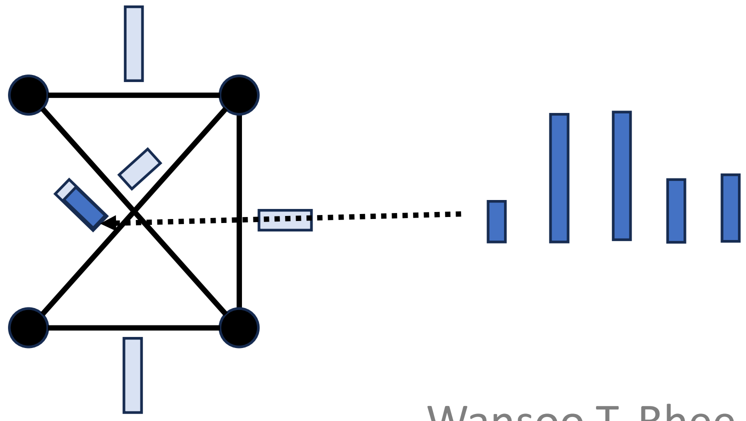

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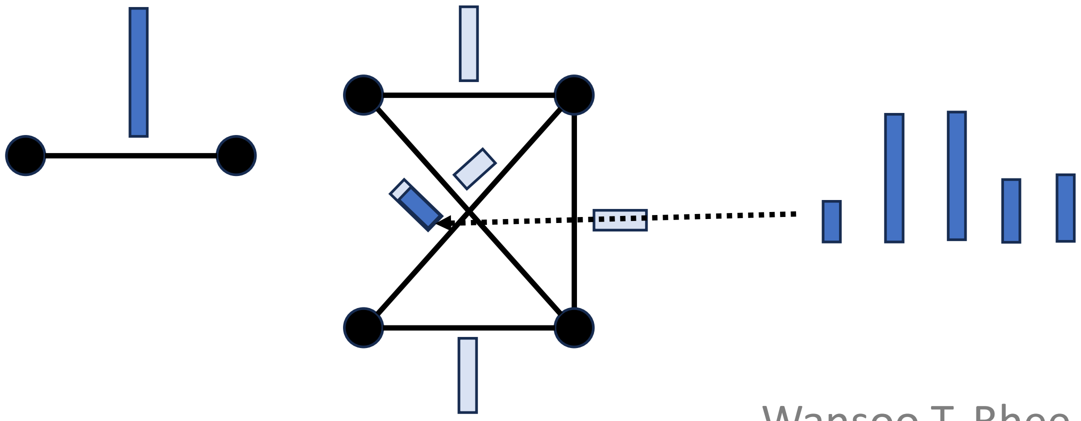
Theorem (Monotone Matching): Given sequences $X_1, \dots, X_n$ and $Y_1, \dots, Y_n$ (arriving online) drawn i.i.d. from the same distribution, we can compute a matching from Y's to X's such that w.h.p.:

- If Y_i is matched to X_j, then $Y_i \leq X_j$
- At most $o(n)$ of the Y's are unmatched

How to use template:

- Compute monotone matching from n' next online arrivals to the realized first n'
- If matched, then assign arrival to corresponding slot in template
- Else, assign arrival to its own separate edge



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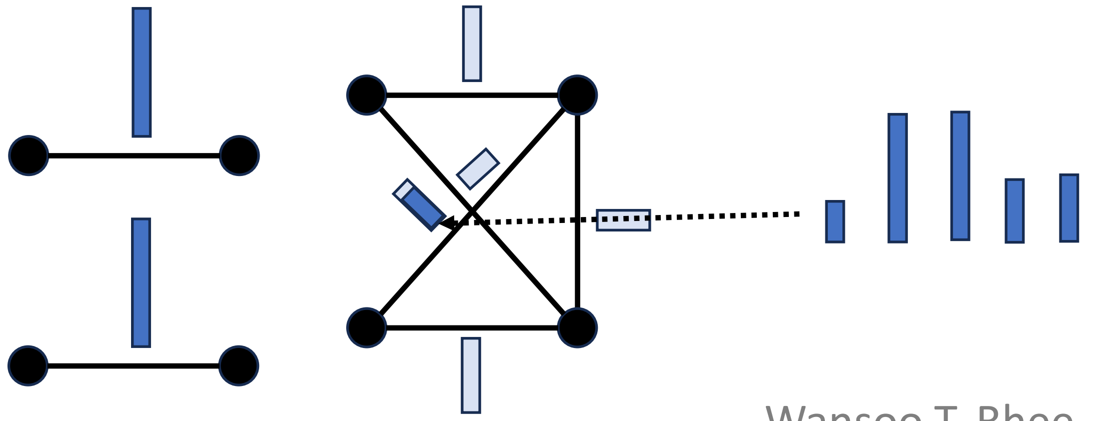

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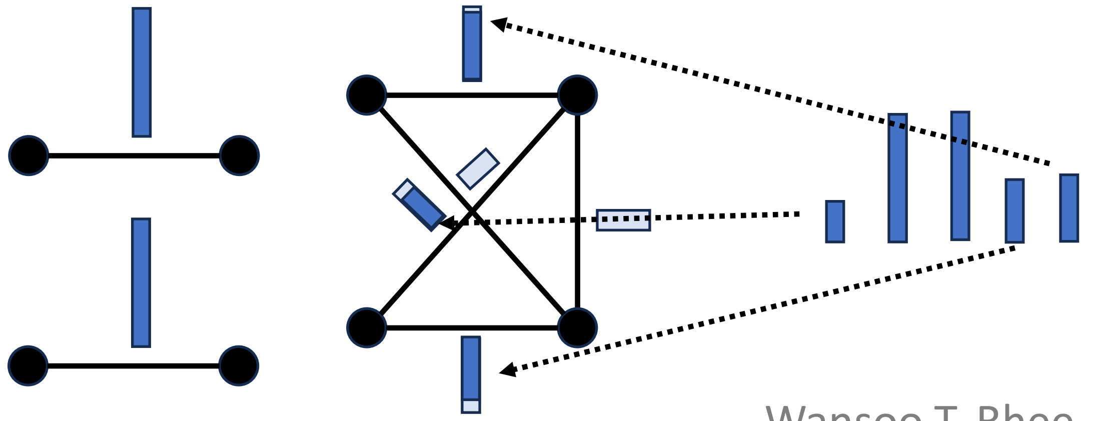

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Conclusion

- Introduce Online Demand Scheduling with Failover
- **Worst Case**
  - Competitive ratio $\rightarrow \frac{1}{2}$ as $m \rightarrow \infty$
  - Tight lower bound
  - Reserve cliques for different sizes
- **Stochastic i.i.d.:**
  - Competitive ratio $\rightarrow 1$ as $m \rightarrow \infty$
  - Learn from past arrivals using template assignments
  - Monotone matching theorem