The Power of Migrations in Dynamic Bin Packing

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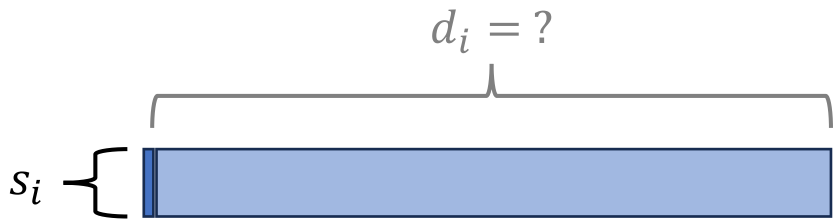
Microsoft Research Carnegie Mellon ⇒ Microsoft

To appear in Sigmetrics 2025

Dynamic Bin Packing



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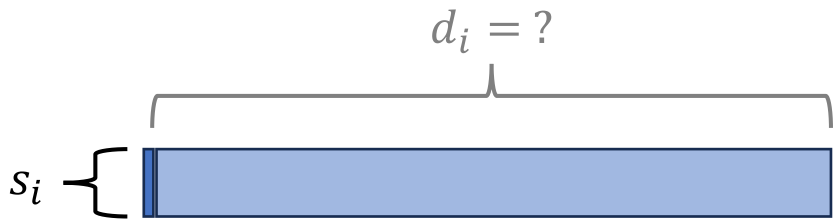

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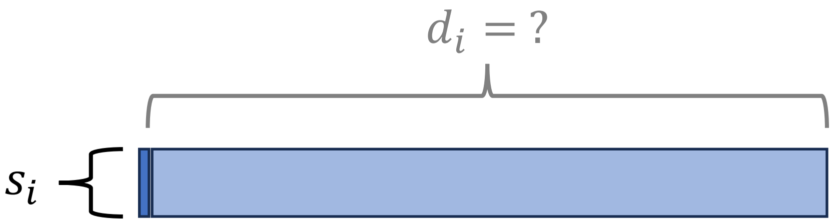

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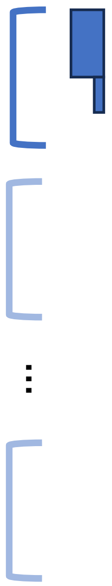

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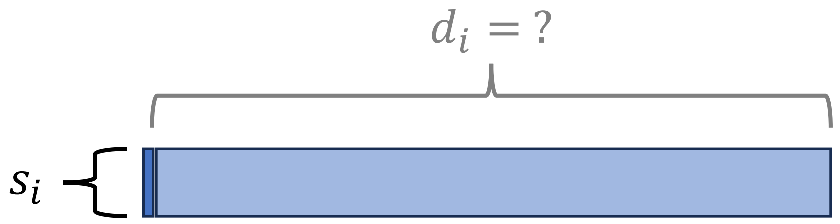

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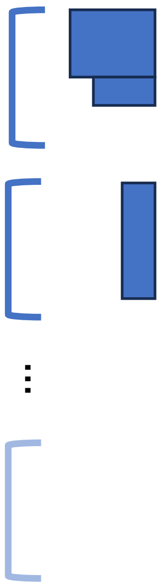

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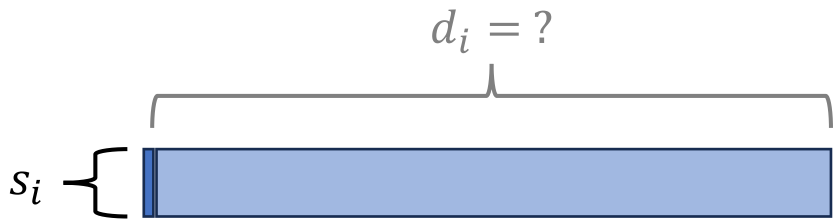

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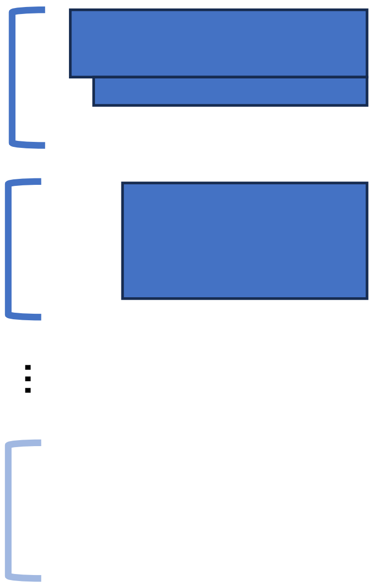

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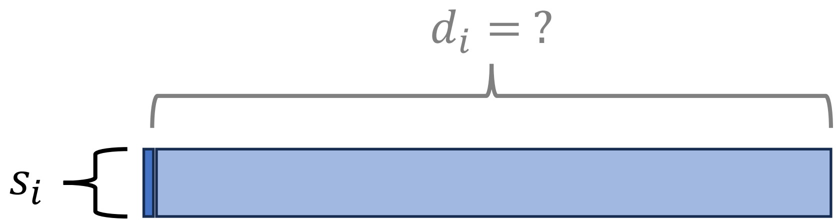

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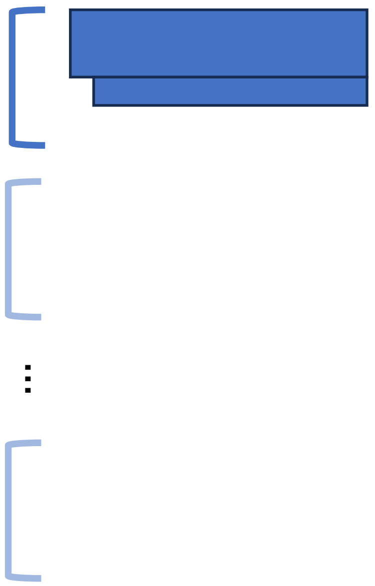

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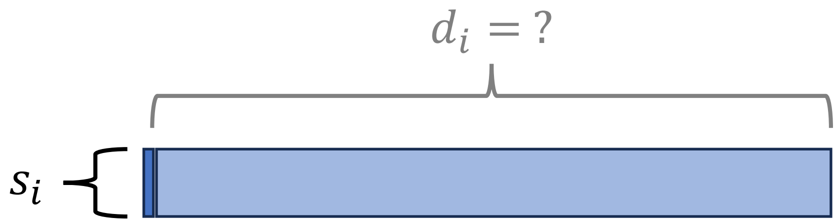

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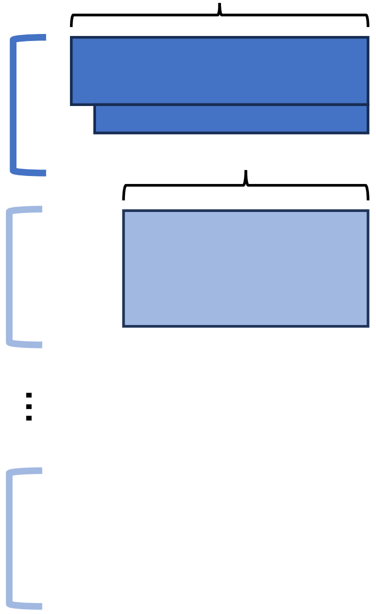

Minimize total active time over all bins

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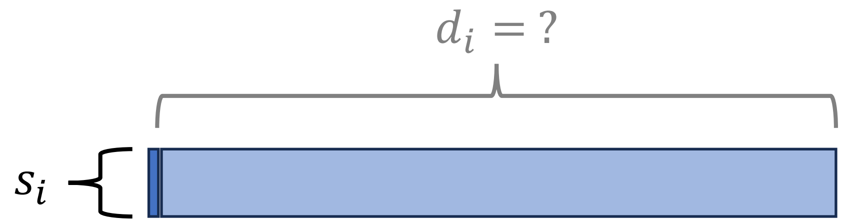

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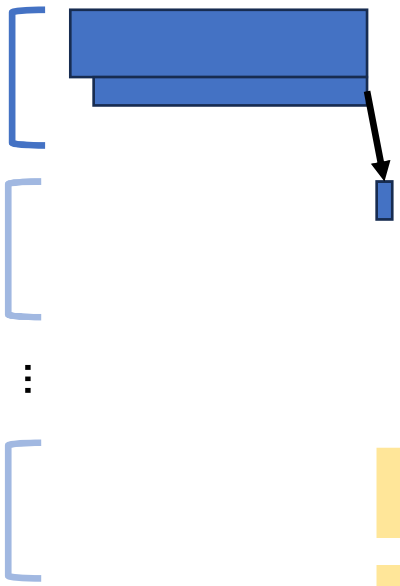

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Can also migrate items

Related Work

$$\mu = \frac{\max duration}{\min duration}$$

$$n = \#\, items$$

- **No migrations:** $\Theta(\mu)$-approximation (first fit)

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  On dynamic bin packing for resource allocation in the cloud. SPAA 2014

- $> \boldsymbol{n}$ **migrations:** $\approx 1.387 + \epsilon$-approximation with $O(\frac{n}{\epsilon^2})$ migrations

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- Can get better guarantees with no migrations if know item durations exactly or approximately (predictions)

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  Dynamic Bin Packing with Predictions. SIGMETRICS 2023

  Niv Buchbinder, Yaron Fairstein, Konstantina Mellou, Ishai Menache, Joseph (Seffi) Naor:
  Online Virtual Machine Allocation with Lifetime and Load Predictions. SIGMETRICS 2021

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  Niv Buchbinder, Ya
  Online Virtual Mac

What can we do with $< n$ migrations? ϵn? $\sqrt{n}$?

Bad Example

- μ^2 items; all arriving at time 0 with size $\frac{1}{\mu}$
  - μ of them are long with duration μ
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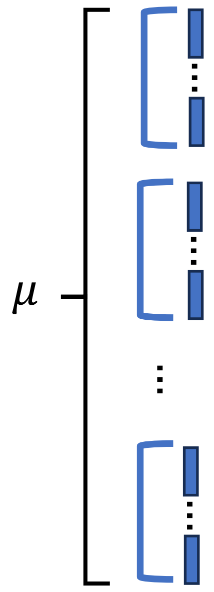

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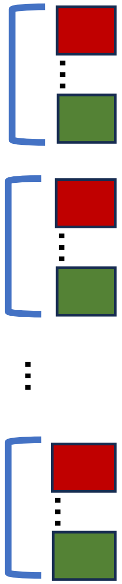

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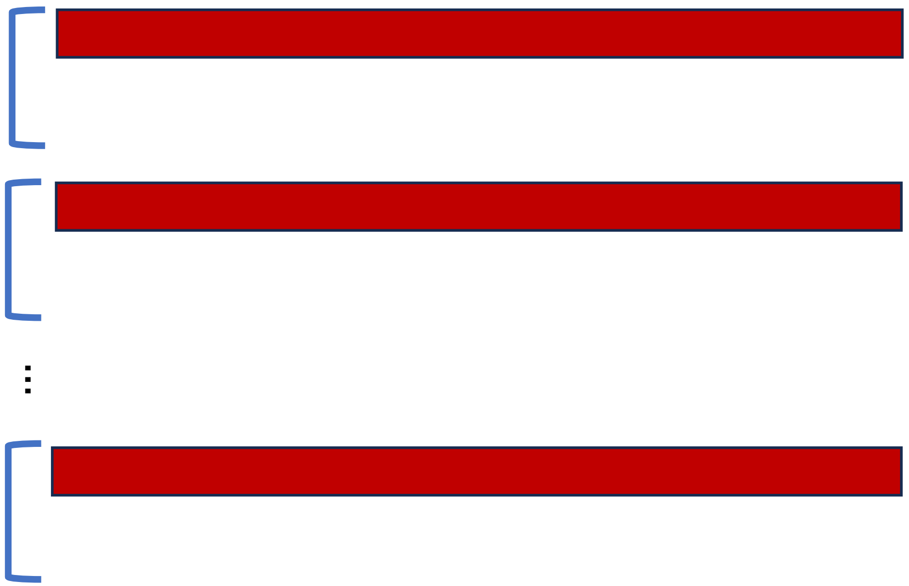

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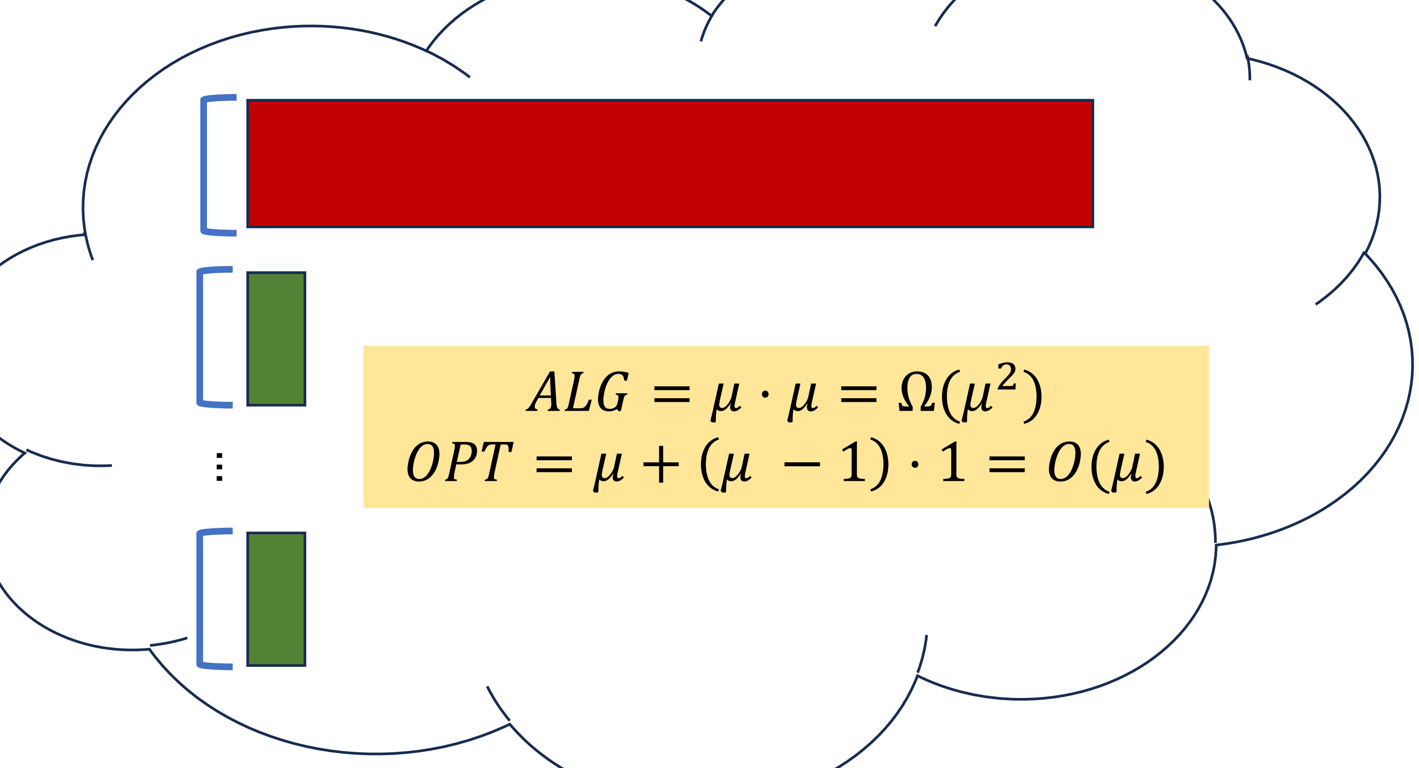

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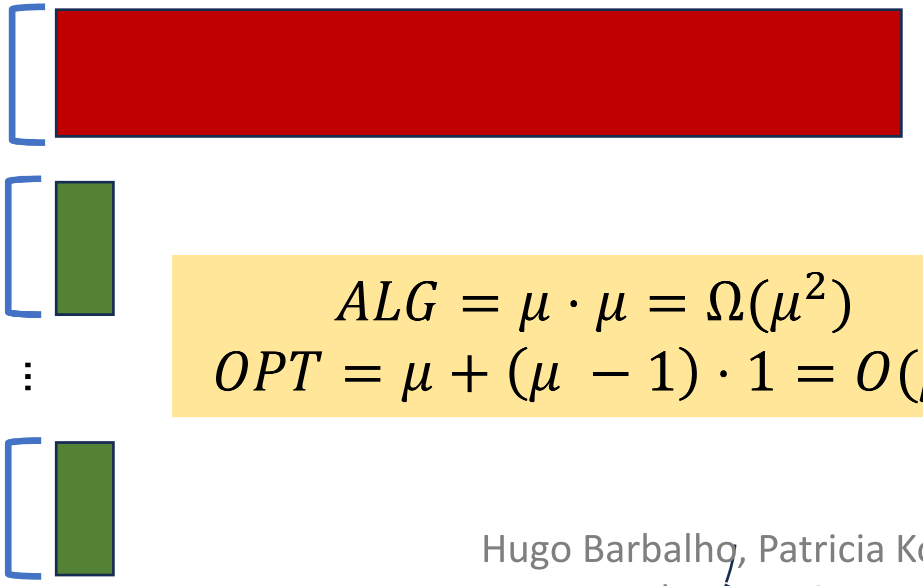

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This actually happens in practice

Hugo Barbalho, Patricia Kovaleski, Beibin Li, Luke Marshall, Marco Molinaro, Abhisek Pan, Eli Cortez, Matheus Leao, Harsh Patwari, Zuzu Tang, Larissa Rozales Gonçalves, David Dion, Thomas Moscibroda, Ishai Menache: Virtual Machine Allocation with Lifetime Predictions. MLSys 2023

Our Results (Part 1)

- **Sublinear migrations:** Any algorithm that does $o(n)$ migrations must be $\Omega(\mu)$-approximate
- $< \boldsymbol{n}$ **migrations:** For any $\epsilon \in (0,1)$, can get $\approx \frac{1}{\epsilon}$-approximation using ϵn migrations, and this is best possible*

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- Want $\approx \frac{1}{\epsilon}$-approximation $\Rightarrow$ suffices to ensure most bins are $\geq \epsilon$-full
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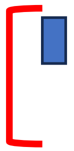
Initially bad



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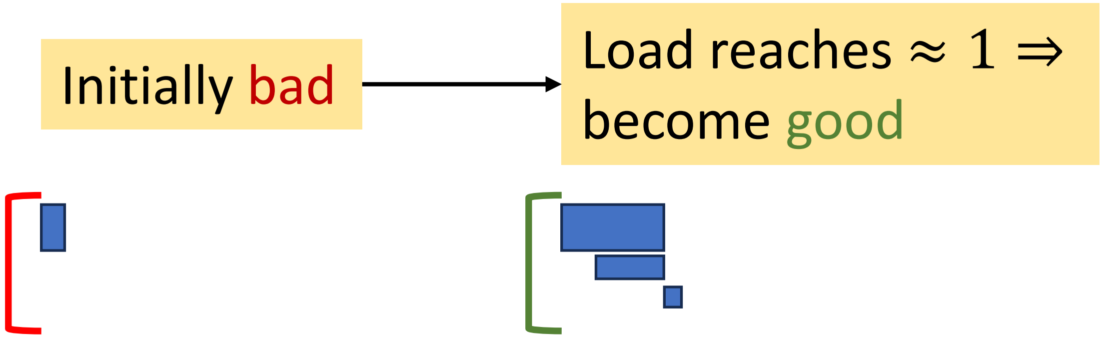

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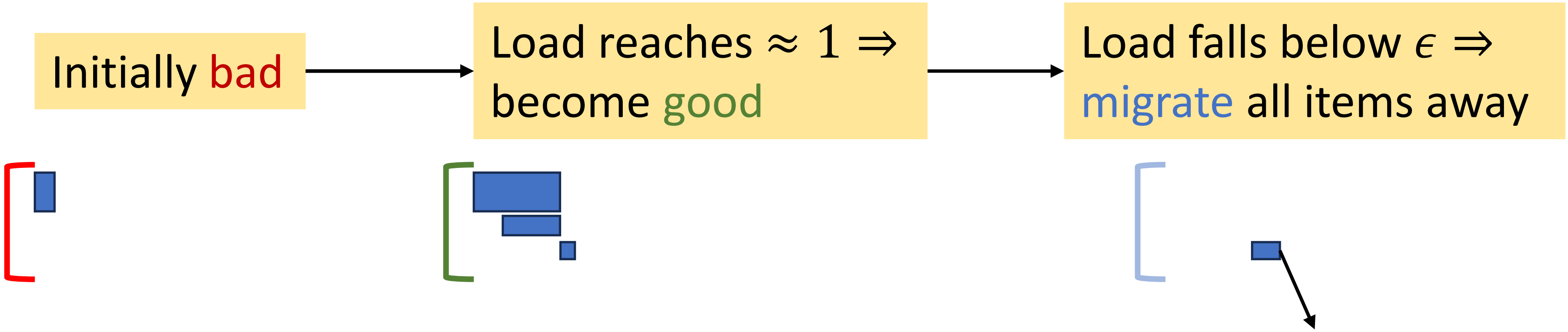

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Initially bad → Load reaches $\approx 1 \Rightarrow$ become good → Load falls below $\epsilon \Rightarrow$ migrate all items away

- Can ensure ≤ 1 bad bins at any time
- Migrate ϵ-fraction of bin load when $1 - \epsilon$-fraction departs

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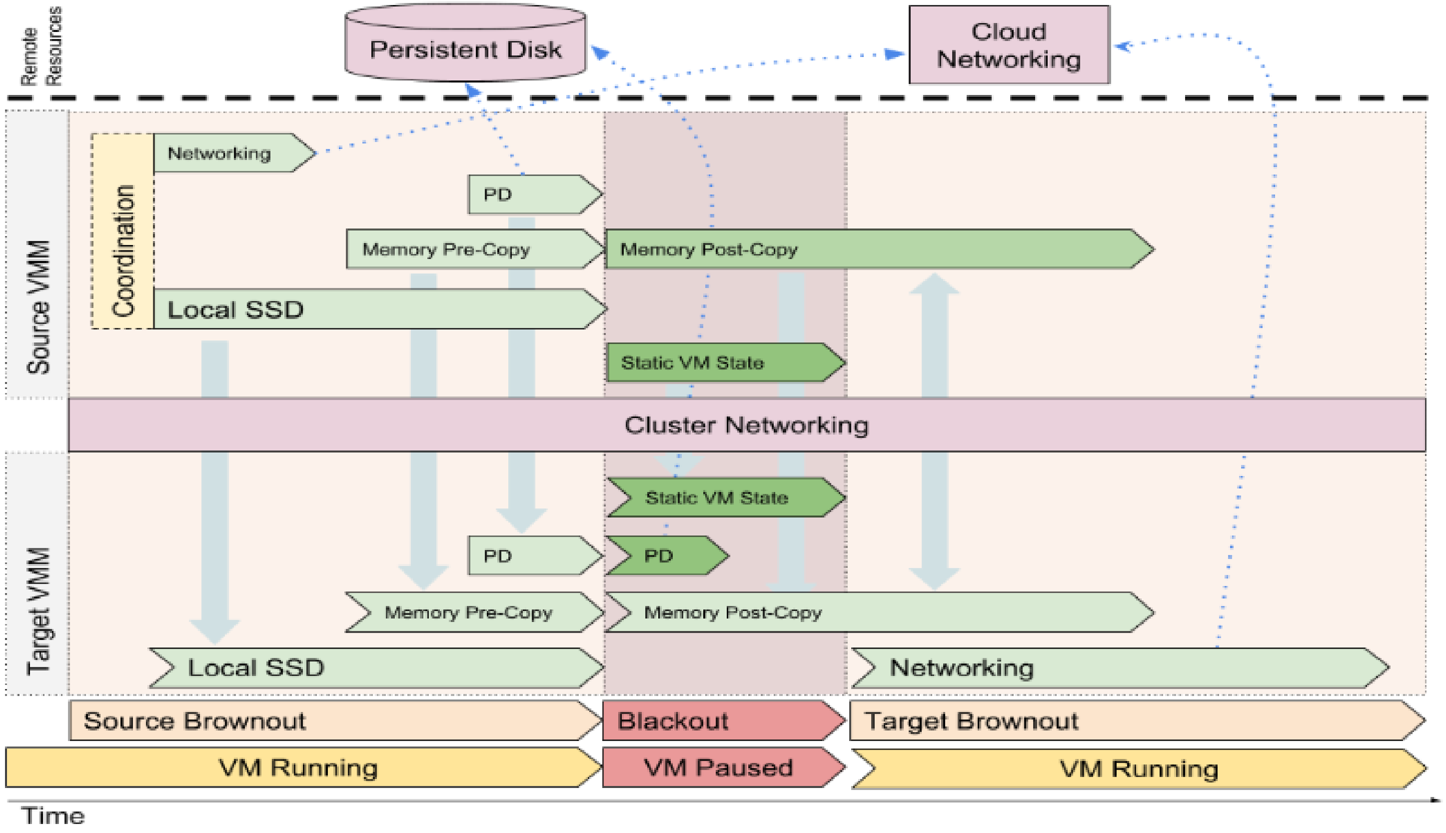
Real cost of migration



Adam Ruprecht, Danny Jones, Dmitry Shiraev, Greg Harmon, Maya Spivak, Michael Krebs, Miche Baker-Harvey, Tyler Sanderson: VM Live Migration At Scale. VEE 2018

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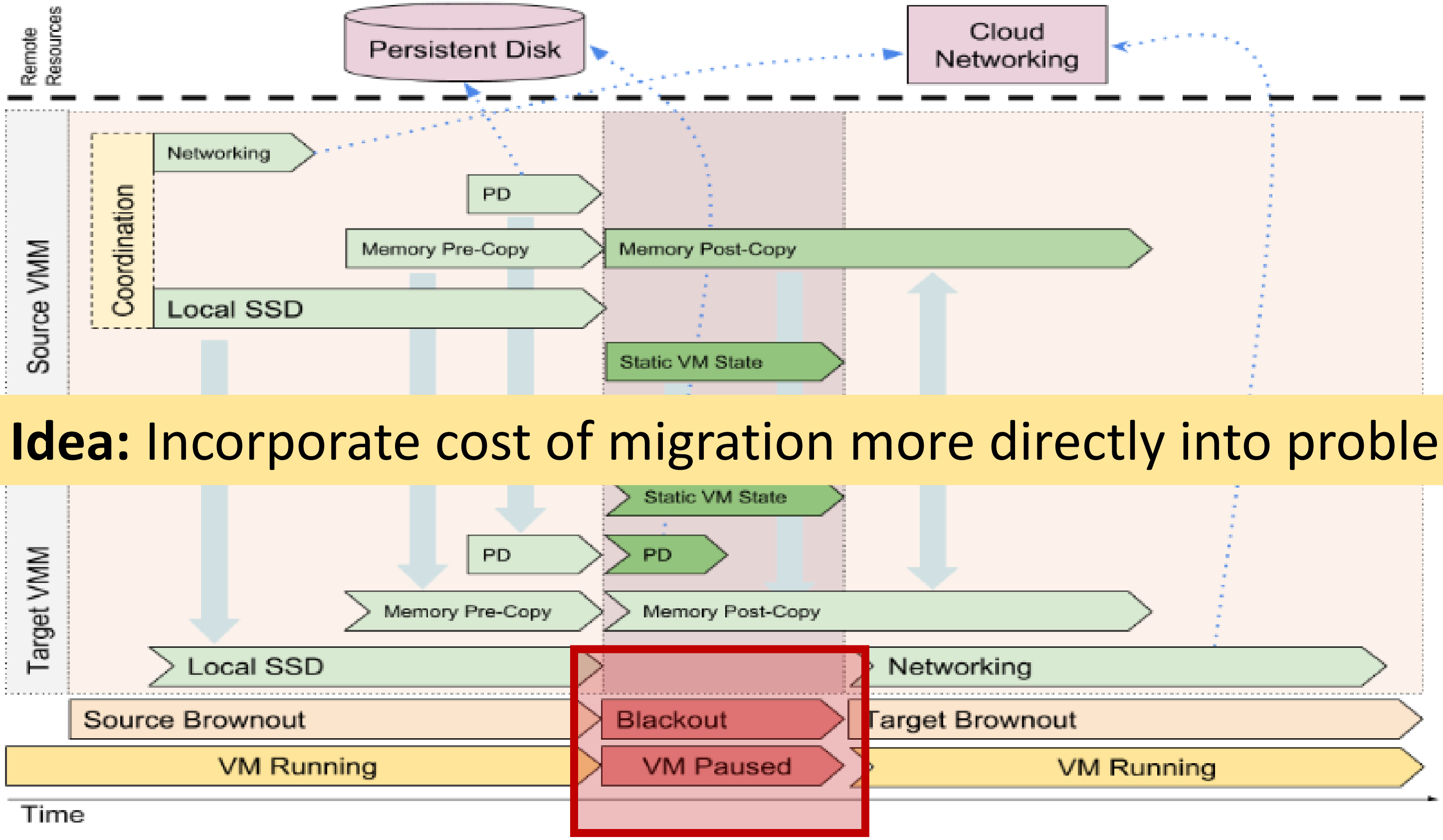

Idea: Incorporate cost of migration more directly into problem

Adam Ruprecht, Danny Jones, Dmitry Shiraev, Greg Harmon, Maya Spivak, Michael Krebs, Miche Baker-Harvey, Tyler Sanderson: VM Live Migration At Scale. VEE 2018

Dynamic Bin Packing with Delays

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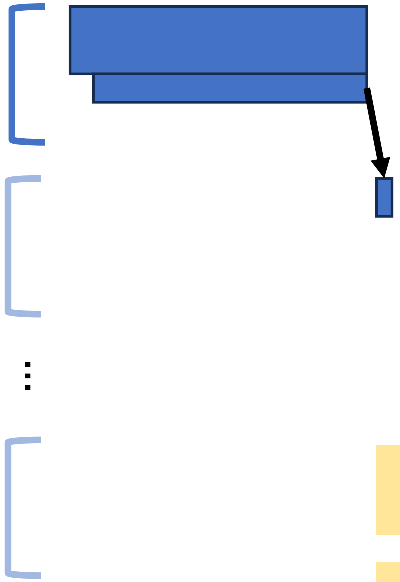

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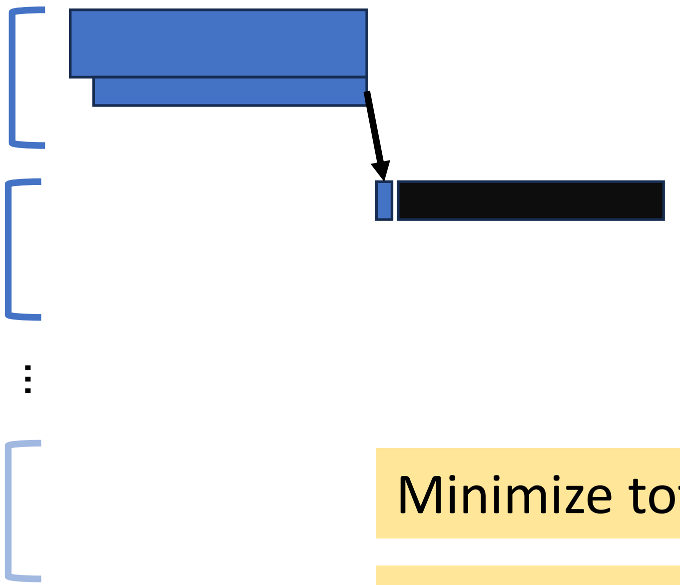

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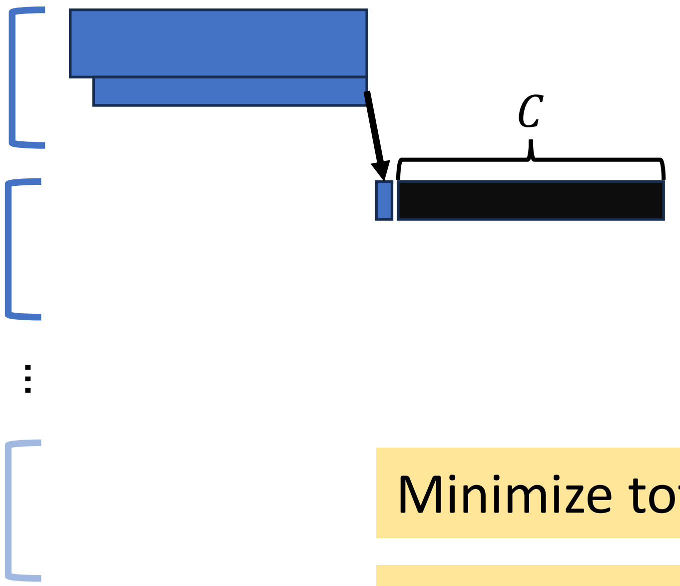

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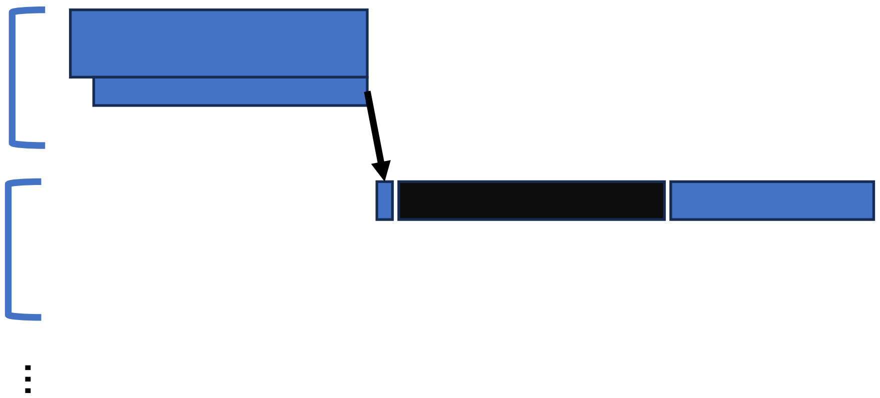

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- Best of both worlds: never worse than doing no migrations, but can be much better
- In practice, the minimum duration can be ≈ 1 millisecond, and the maximum ≈ 1 year ⇒ $\mu \approx 10^{10}$
- However, migrating an item incurs a delay of ≈ 1 second ⇒ $C \approx 10^3$

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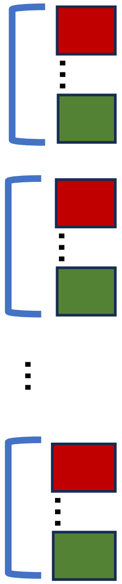

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First Try: Migrate at time C

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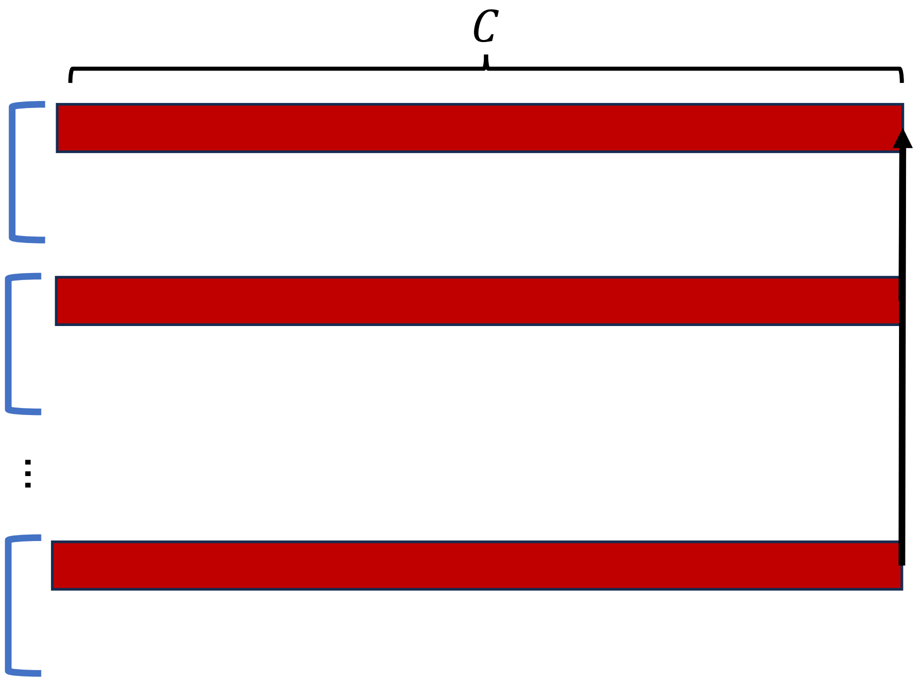

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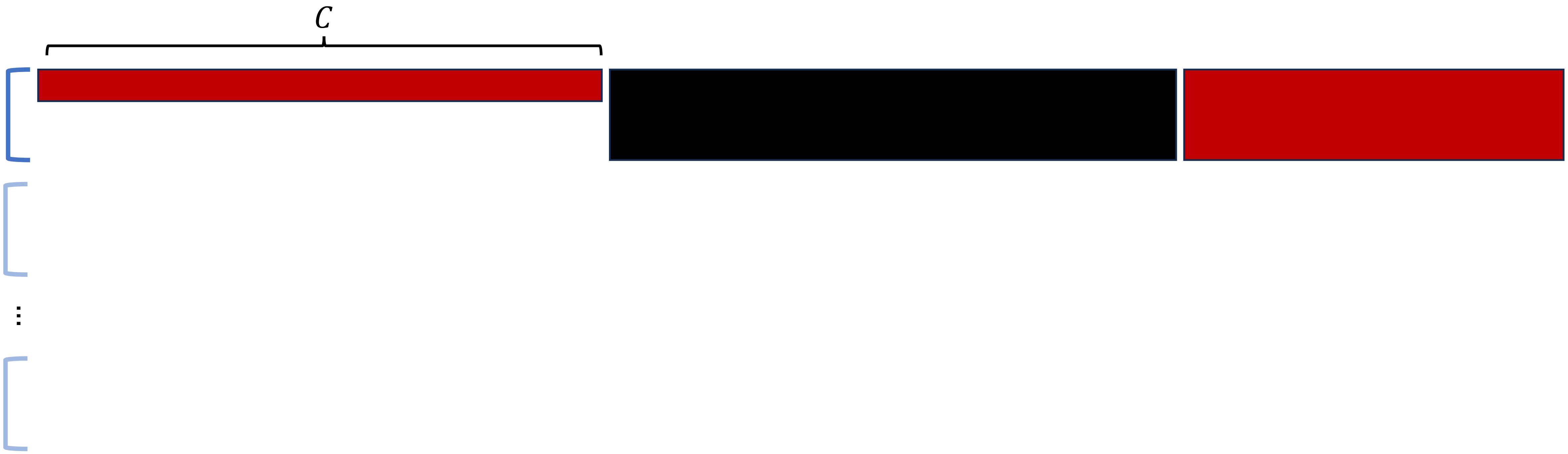

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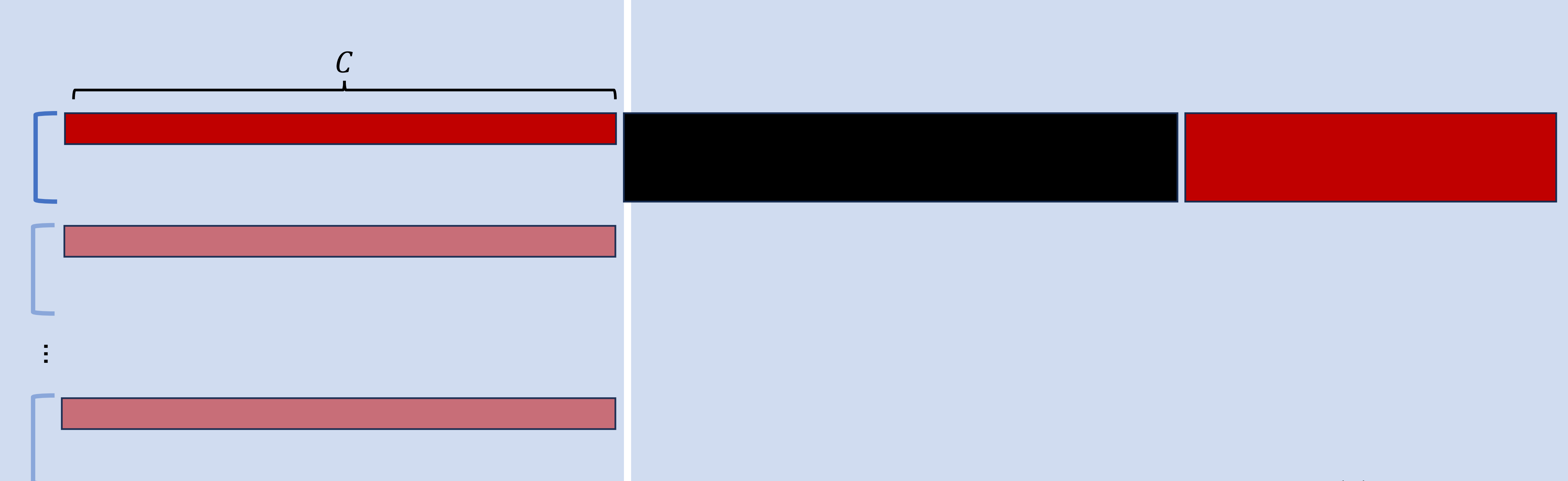

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Good assignment, but increased durations by $O(1)$-factor

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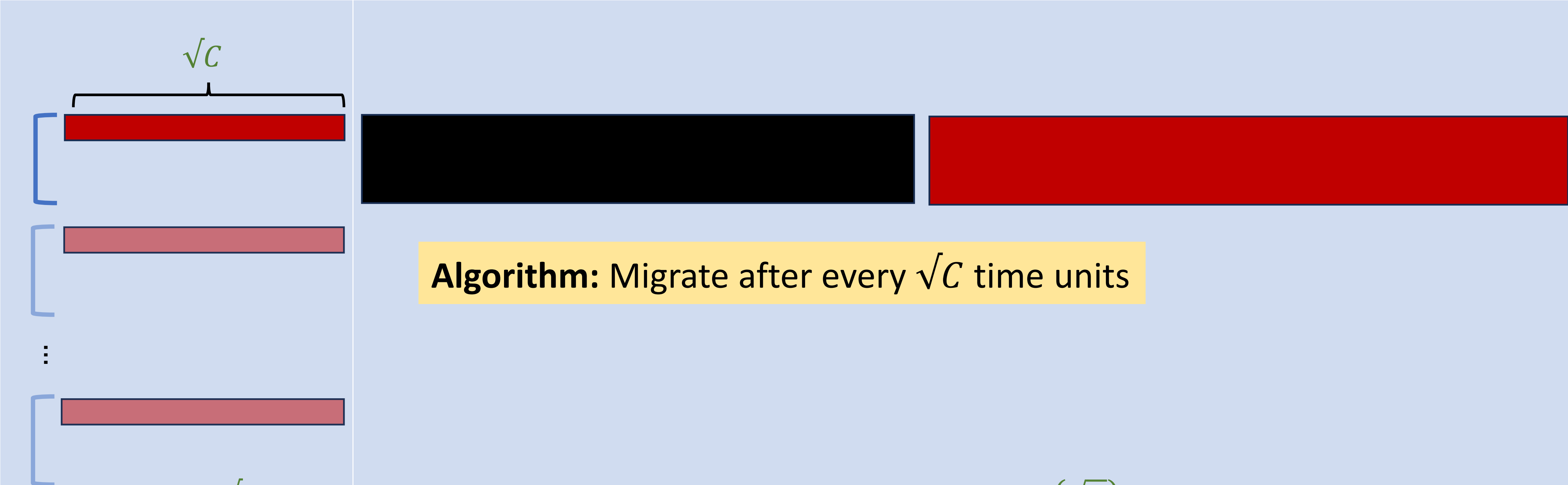

Bad assignment for $\sqrt{C}$ time units

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- However, migrating an item incurs a delay of ≈ 1 second ⇒ $C \approx 10^3$

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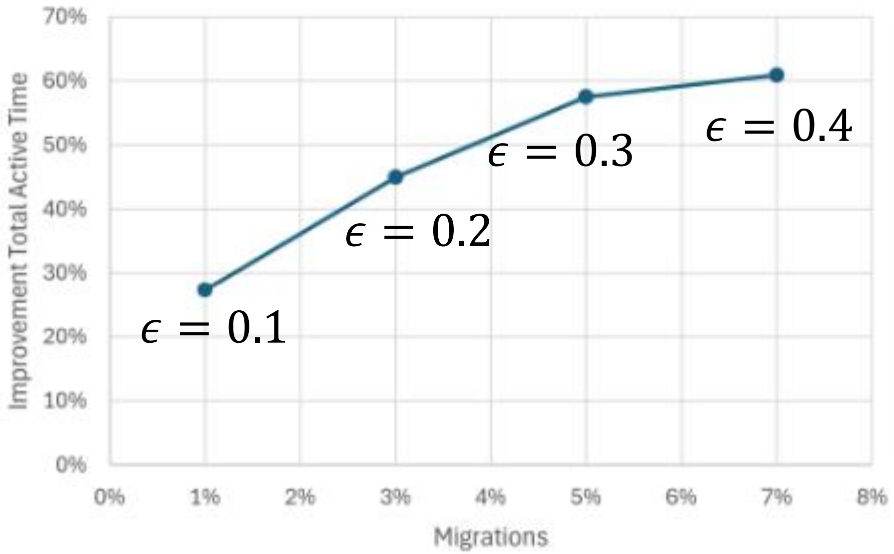
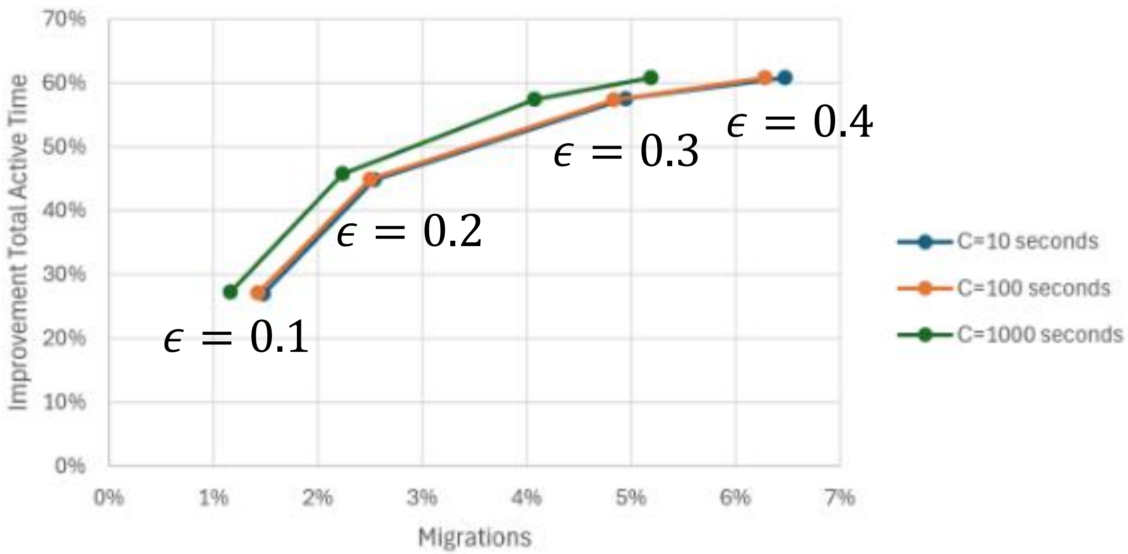
Experiments

- Data from Microsoft Azure (VM requests)
- Combine both $< n$ migration and delay algorithms (classify bins as bad and good; only migrate items after every C time units)

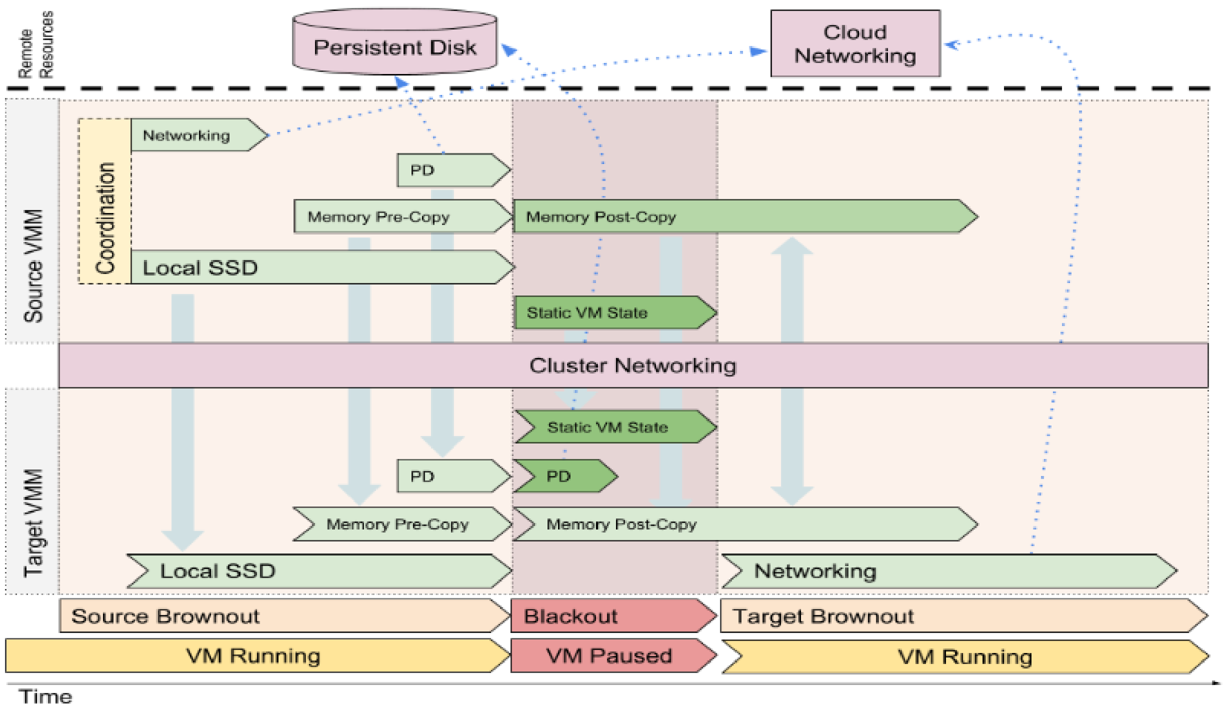




Ori Hadary, Luke Marshall, Ishai Menache, Abhisek Pan, Esaias E. Greeff, David Dion, Star Dorminey, Shailesh Joshi, Yang Chen, Mark Russinovich, Thomas Moscibroda: Protean: VM Allocation Service at Scale. OSDI 2020

Conclusion

- Fill gaps in our understanding for $o(n)$ and ϵn migrations
- Introduce delays to dynamic bin packing



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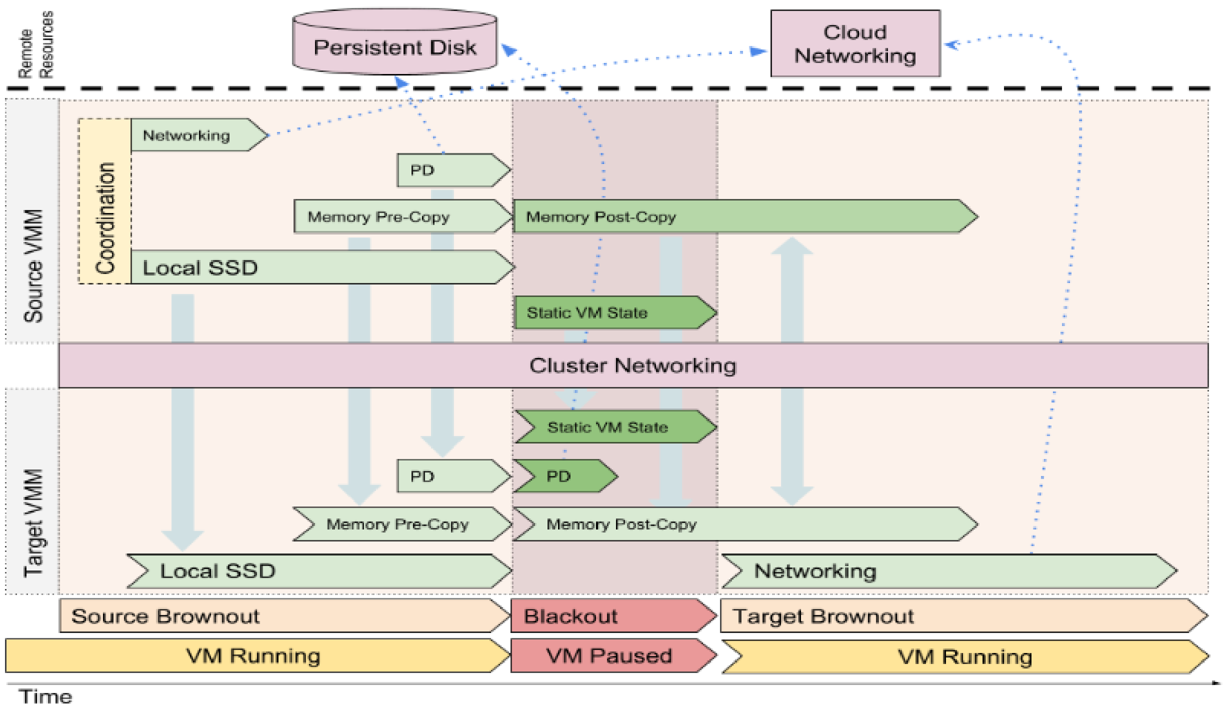

Open Questions:

- Remove additive $\log n$ in ϵn migration case
- Beyond worst case model
- Consider networking limits